



# Online Quantitative Timed Pattern Matching with Semiring- Valued Weighted Automata

Masaki Waga

Kyoto University

12 May 2021, YR-OWLS

Based on the paper at FORMATS'19



Monitoring

# Online Quantitative Timed Pattern Matching with Semiring- Valued Weighted Automata

Masaki Waga

Kyoto University

12 May 2021, YR-OWLS

Based on the paper at FORMATS'19

# Safety Critical CPSs

## Self-driving car crash in Arizona: Red light runner hits Waymo van



Copyright 2018 Scripps Media, Inc. All rights reserved. This material may not be published, broadcast, rewritten, or redistributed.  
Photo by: Air15

**BBC** Sign in News Sport Reel Worklife Travel Future M

**NEWS**

Home Video World Asia UK Business Tech Science Stories Entertainment &

Technology

### Tesla Model 3: Autopilot engaged during fatal crash

17 May 2019

f Share

A photograph of a red Tesla Model 3 car after a crash. The car is parked on a paved surface, and its front end is severely damaged. The car is surrounded by other vehicles and debris in the background. A black box with the text 'The Tesla Model 3 after the crash' is at the bottom of the image. A small 'NTSB' logo is in the bottom right corner of the image.

<https://www.abc15.com/news/region-southeast-valley/chandler/waymo-car-involved-in-chandler-arizona-crash>

<https://www.bbc.com/news/technology-48308852>

# Monitoring

## Specification:

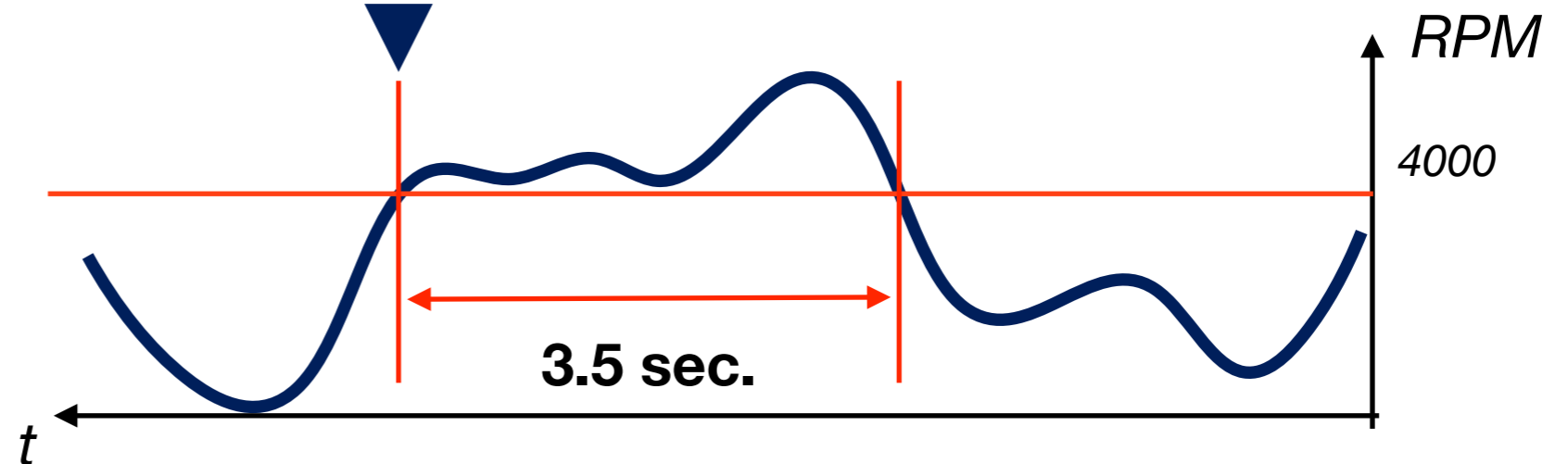
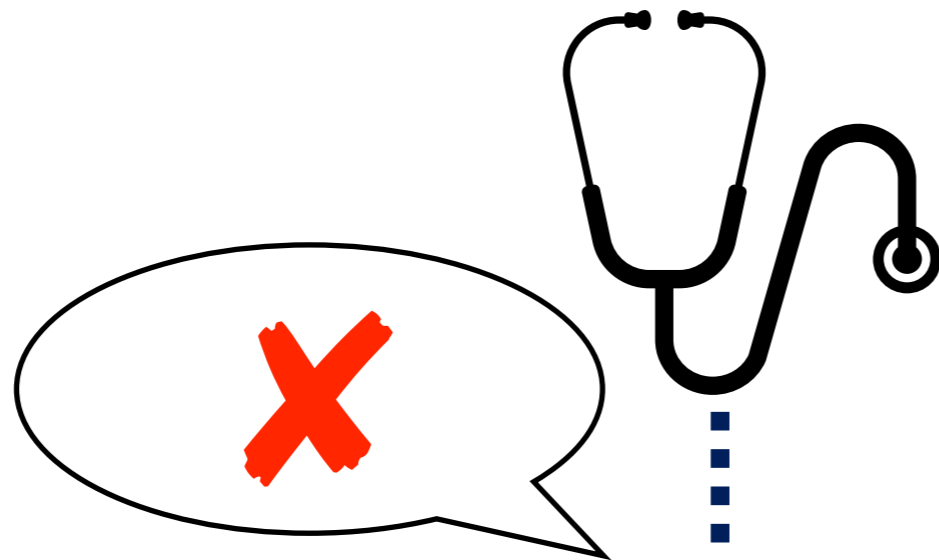
No (RPM > 4000) for > 1 sec.



# Monitoring

## Specification:

No (RPM > 4000) for > 1 sec.



# Timed Pattern Matching

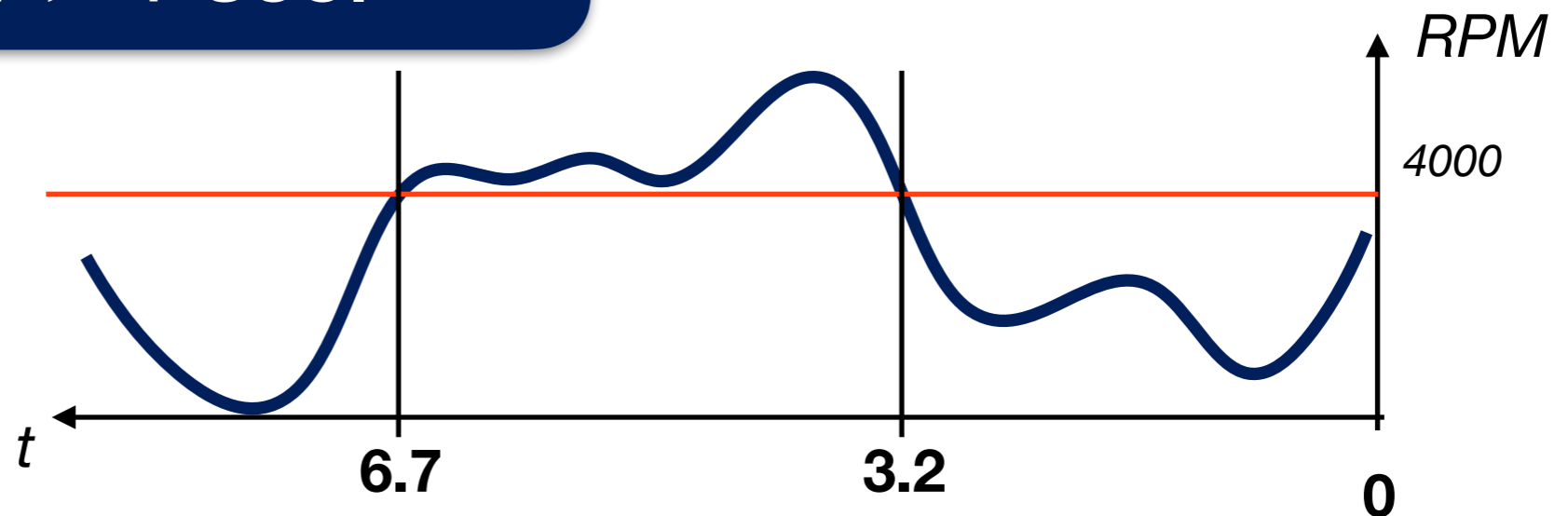
[Ulus+, FORMATS'14]

Given: Signal and Pattern Spec.

Goal: Find all the matching intervals

Pattern Specification:

(RPM > 4000) for > 1 sec.



# Timed Pattern Matching

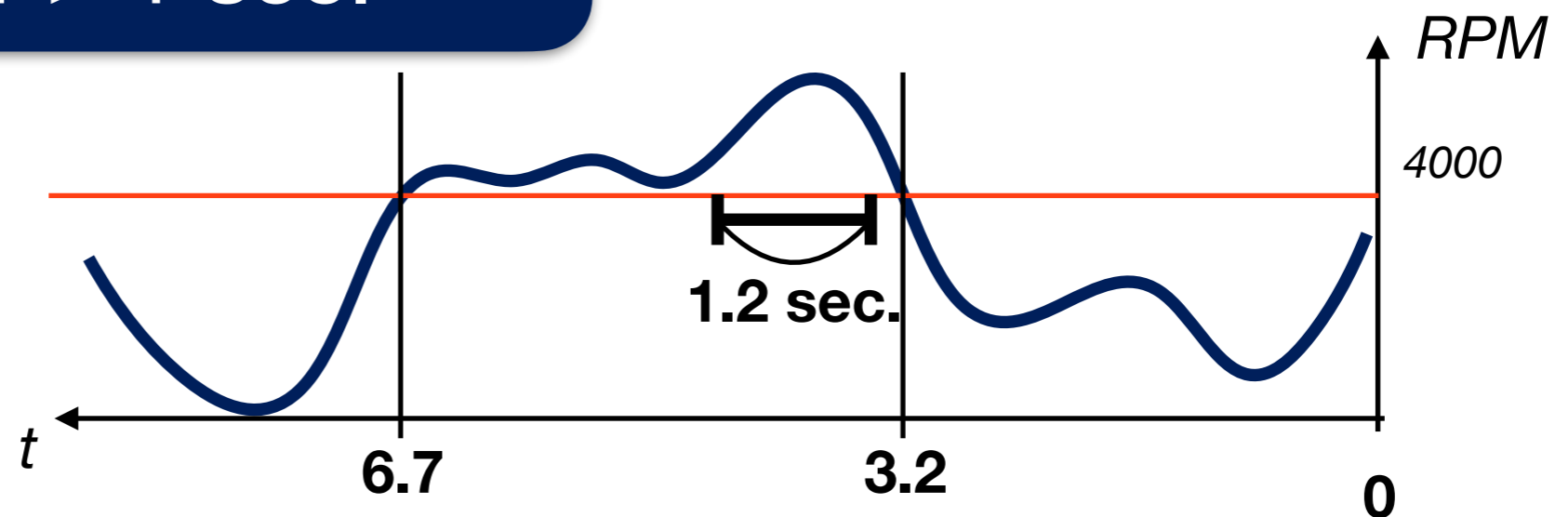
[Ulus+, FORMATS'14]

Given: Signal and Pattern Spec.

Goal: Find all the matching intervals

Pattern Specification:

(RPM > 4000) for > 1 sec.



# Timed Pattern Matching

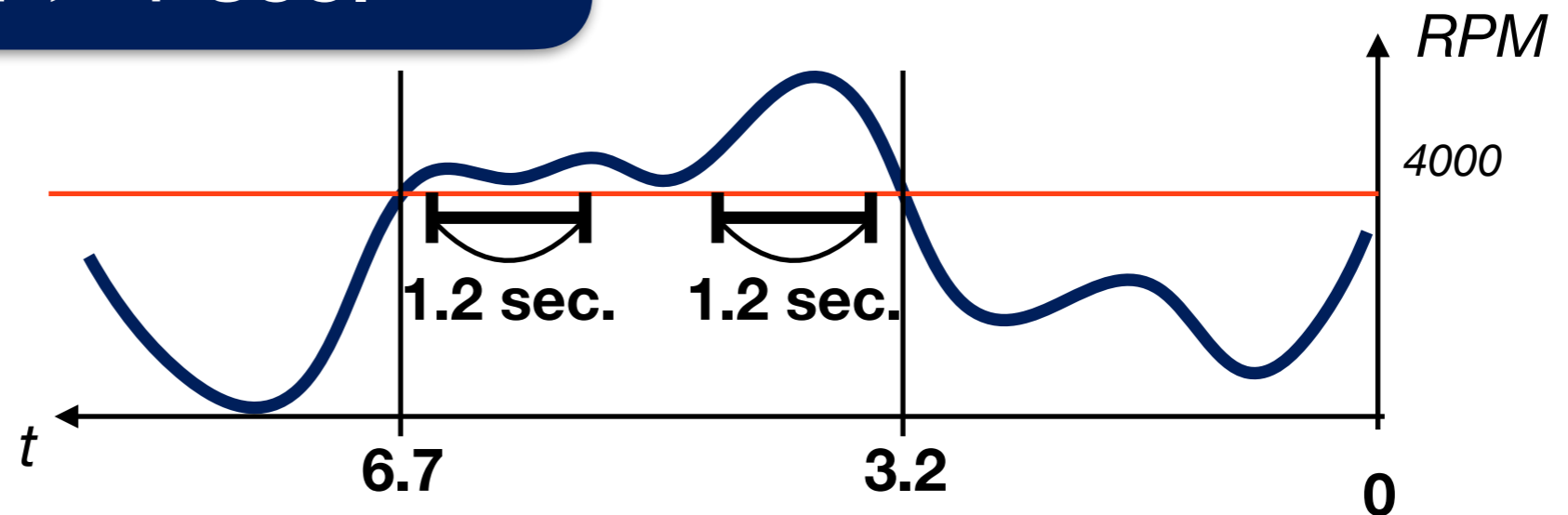
[Ulus+, FORMATS'14]

Given: Signal and Pattern Spec.

Goal: Find all the matching intervals

Pattern Specification:

(RPM > 4000) for > 1 sec.





# Timed Pattern Matching

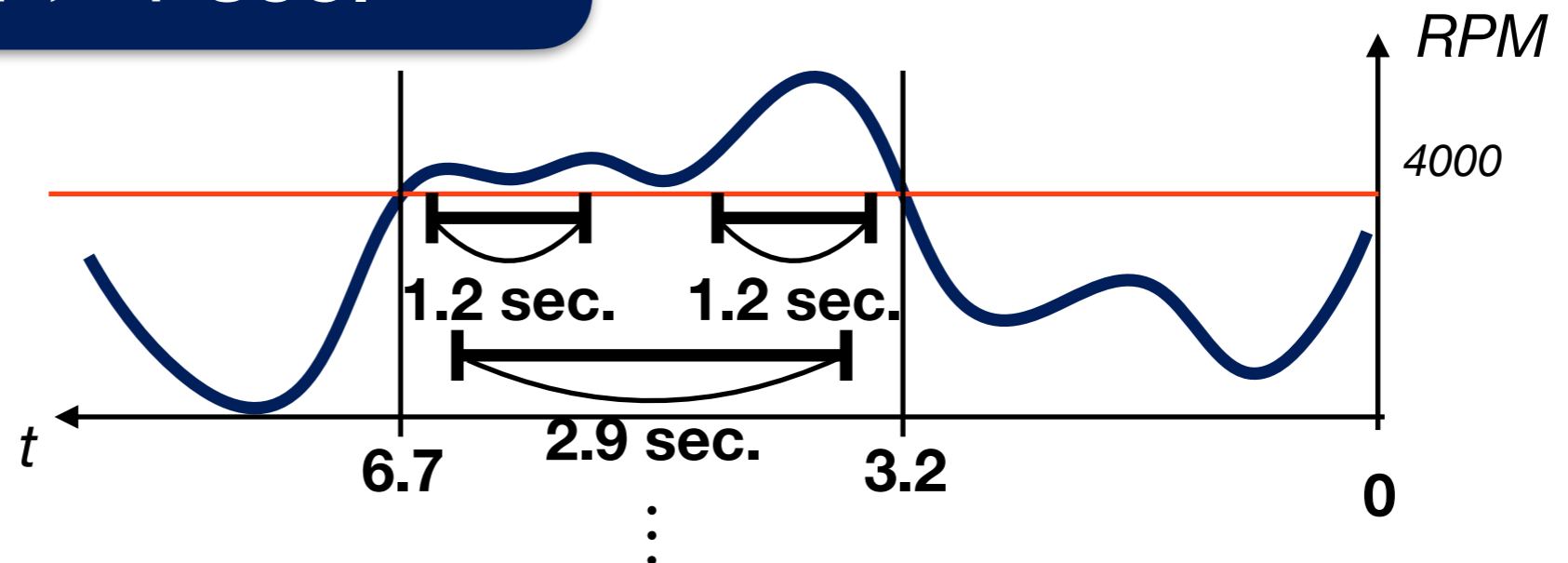
[Ulus+, FORMATS'14]

Given: Signal and Pattern Spec.

Goal: Find all the matching intervals

Pattern Specification:

(RPM > 4000) for > 1 sec.



# Timed Pattern Matching

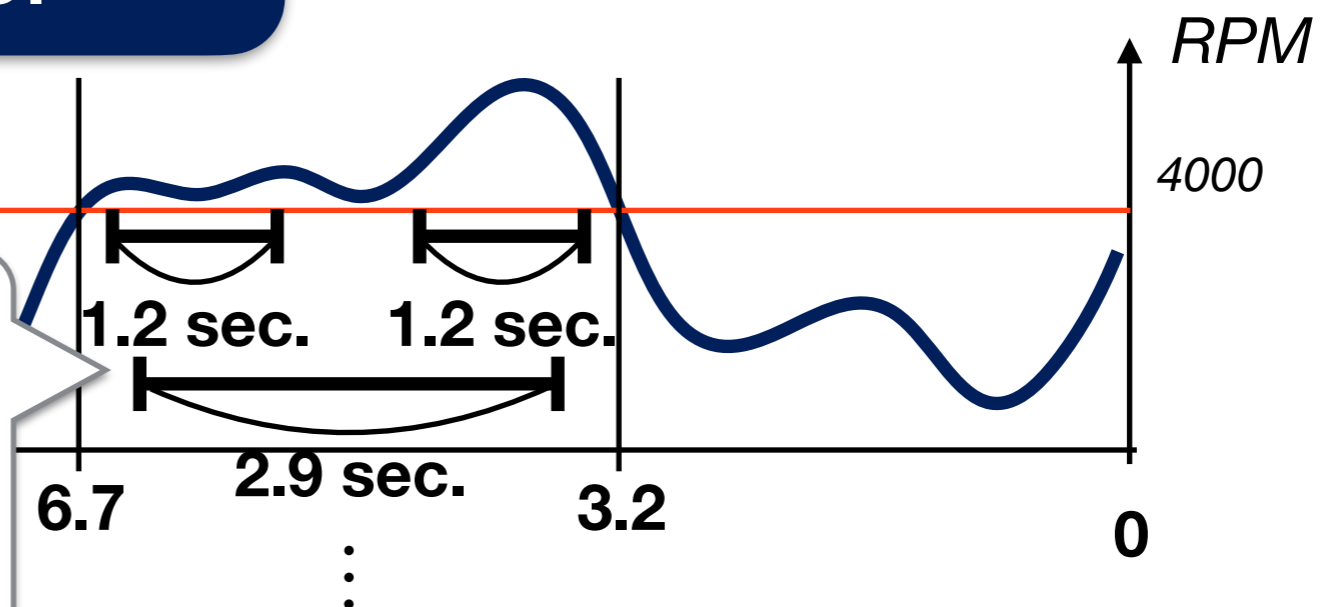
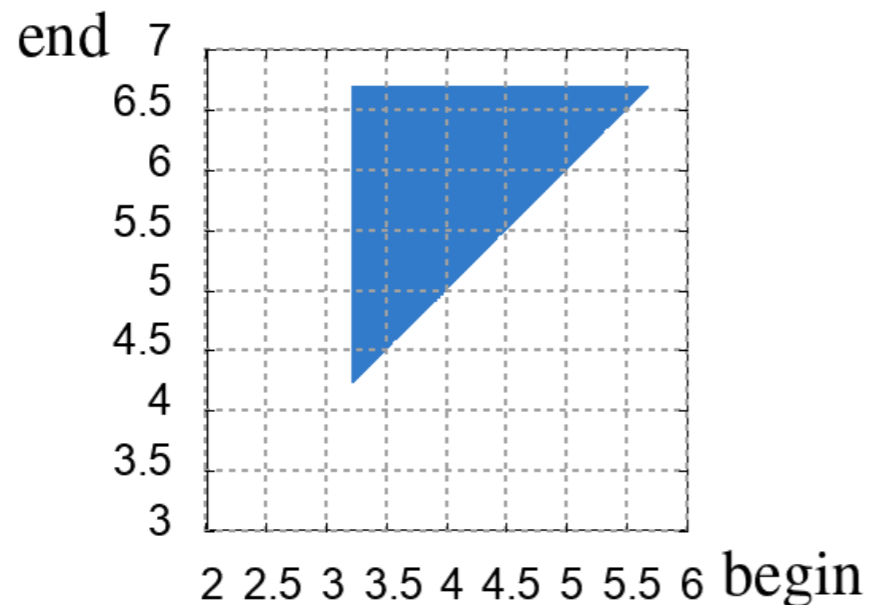
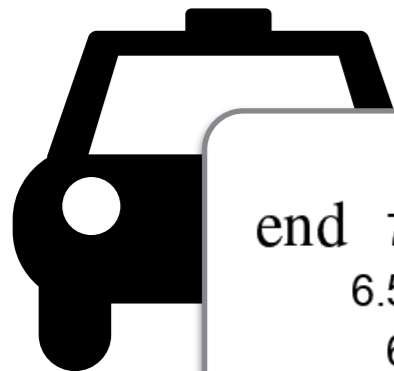
[Ulus+, FORMATS'14]

Given: Signal and Pattern Spec.

Goal: Find all the matching intervals

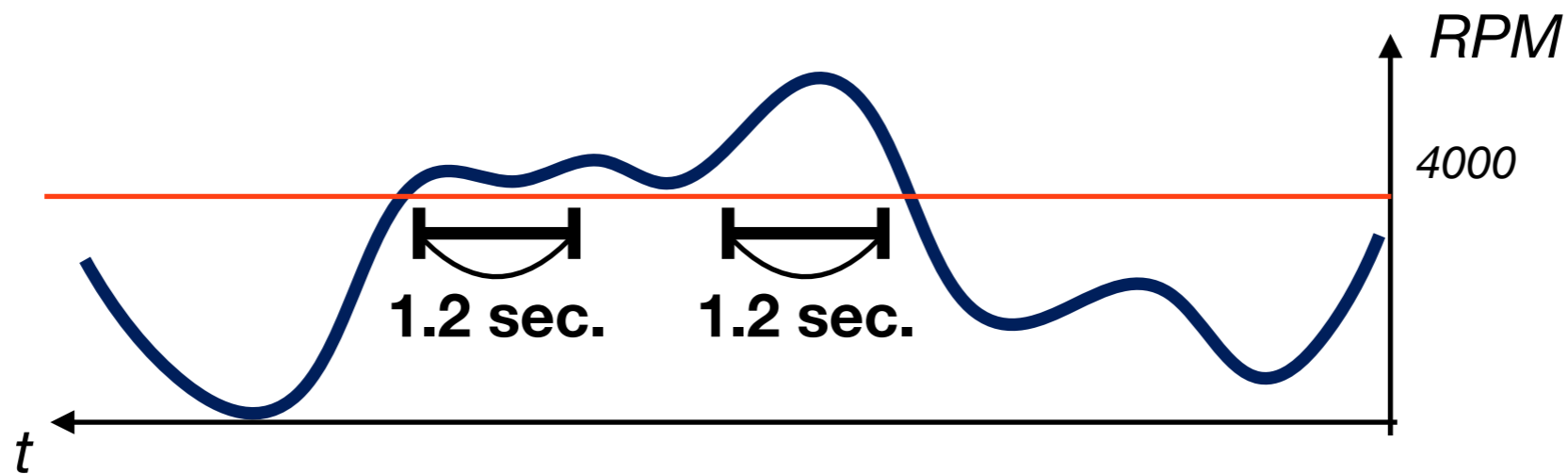
Pattern Specification:

(RPM > 4000) for > 1 sec.



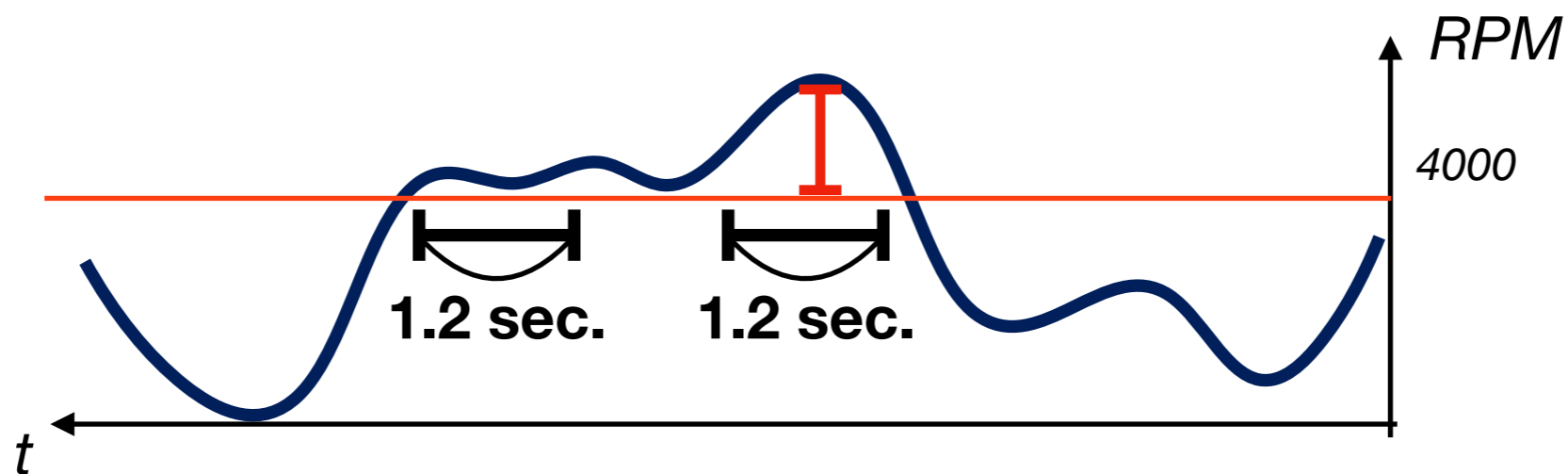
# Qualitative $\rightarrow$ Quantitative

Pattern Specification:  
(RPM  $>$  4000) for  $>$  1 sec.



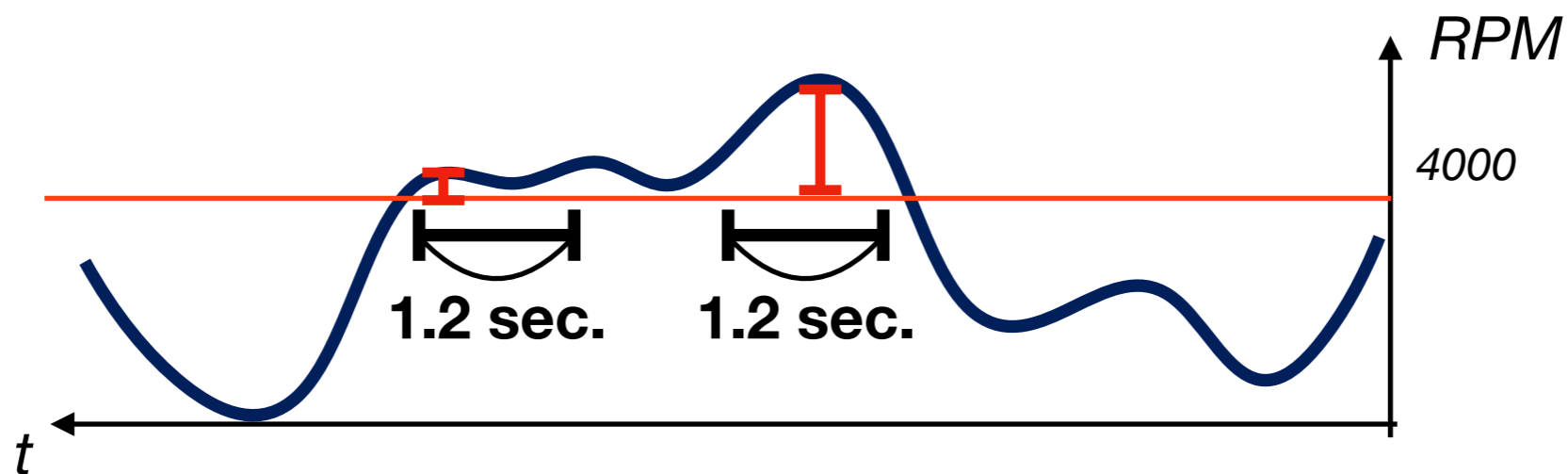
# Qualitative $\rightarrow$ Quantitative

Pattern Specification:  
(RPM  $>$  4000) for  $>$  1 sec.



# Qualitative $\rightarrow$ Quantitative

Pattern Specification:  
(RPM  $>$  4000) for  $>$  1 sec.



# Quantitative Timed Pattern Matching

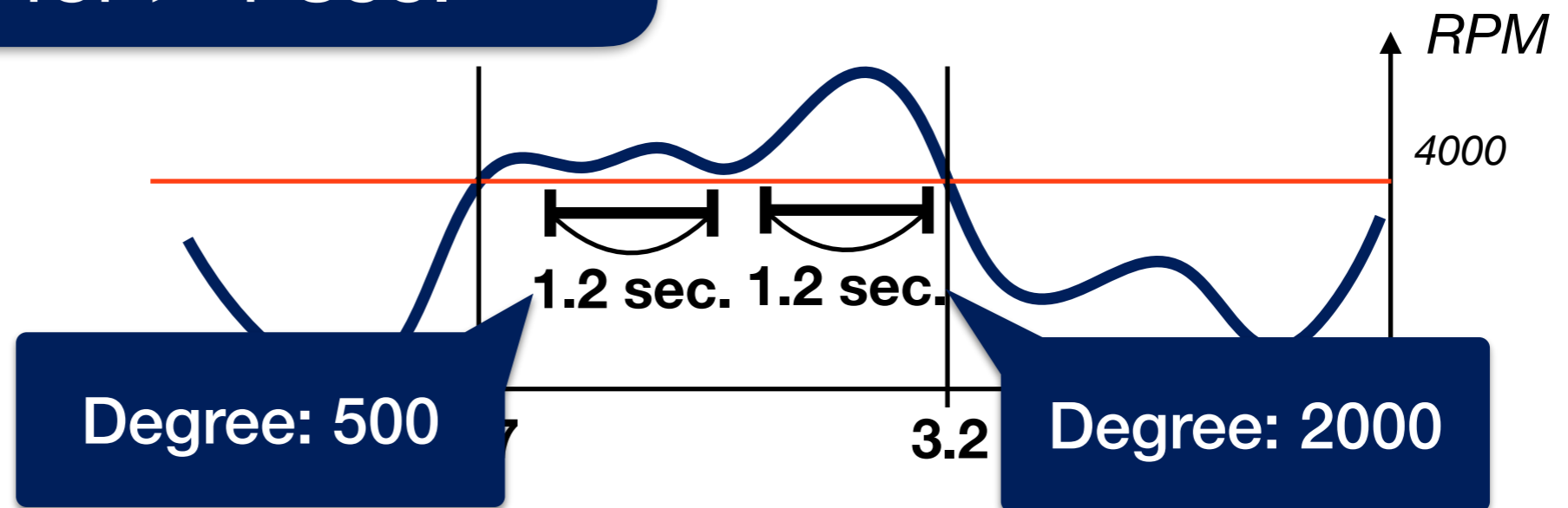
[Bakhirkin+, FORMATS'17]

Given: Signal and Pattern Spec.

Goal: Find all the matching intervals + **satisfaction degree**

Pattern Specification:

(RPM > 4000) for > 1 sec.



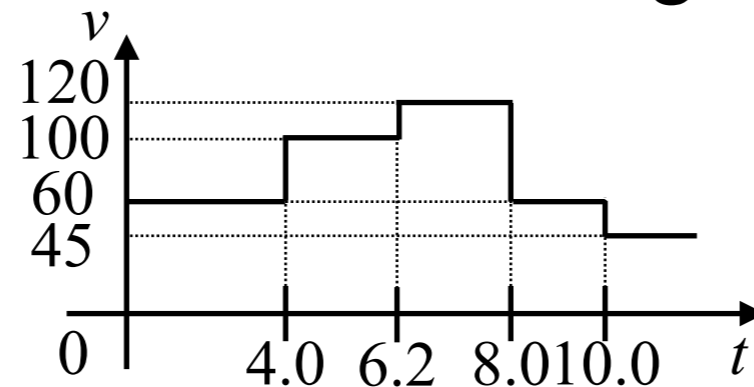
# Quantitative Timed Pattern Matching

## Input

[Bakhirkin+, FORMATS'17]

- **Real-valued piecewise-constant signal  $\sigma$**

- e.g.,



- **Pattern Specification  $\mathcal{W}$**

- Spec. to be monitored

- e.g., The velocity should not keep  $> 80$  for  $> 1$  sec.

## Output

- **Function** assigning the satisfaction degree to each **subsignal**  $\sigma([t,t'])$

- e.g.,  $\mathcal{M}(\sigma, \mathcal{W})(2.0, 4.0) = -20$ ,  $\mathcal{M}(\sigma, \mathcal{W})(6.5, 7.8) = 40$ , ...

satisfaction degree of  $\mathcal{W}$  for  $\sigma([2.0, 4.0))$

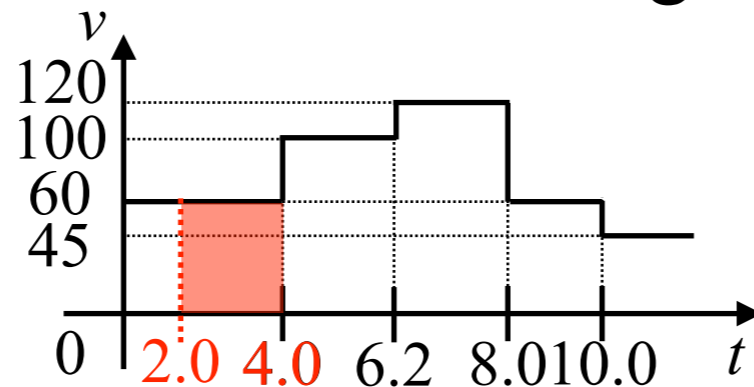
# Quantitative Timed Pattern Matching

## Input

[Bakhirkin+, FORMATS'17]

- **Real-valued piecewise-constant signal  $\sigma$**

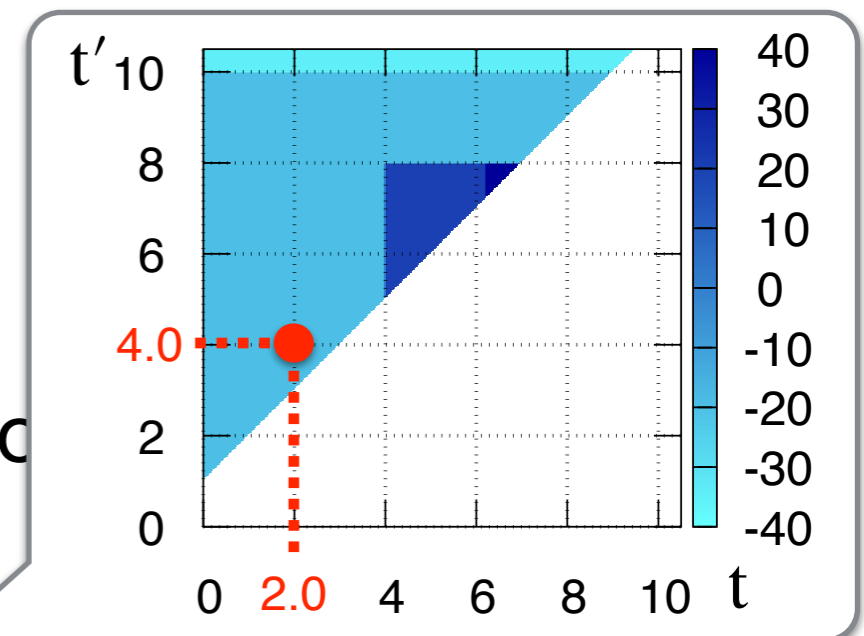
- e.g.,



- **Pattern Specification  $\mathcal{W}$**

- Spec. to be monitored

- e.g., The velocity should not keep  $> 80$  for



## Output

- **Function** assigning the satisfaction degree to each **subsignal**  $\sigma([t, t'])$

- e.g.,  $\mathcal{M}(\sigma, \mathcal{W})(2.0, 4.0) = -20$ ,  $\mathcal{M}(\sigma, \mathcal{W})(6.5, 7.8) = 40$ , ...

satisfaction degree of  $\mathcal{W}$  for  $\sigma([2.0, 4.0))$



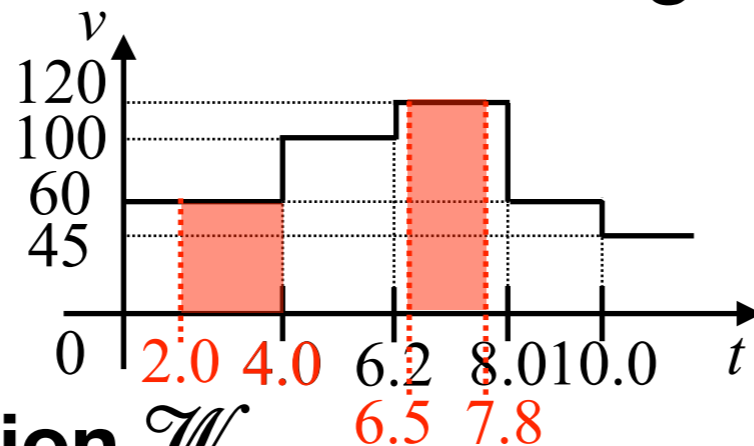
# Quantitative Timed Pattern Matching

## Input

[Bakhirkin+, FORMATS'17]

- **Real-valued piecewise-constant signal  $\sigma$**

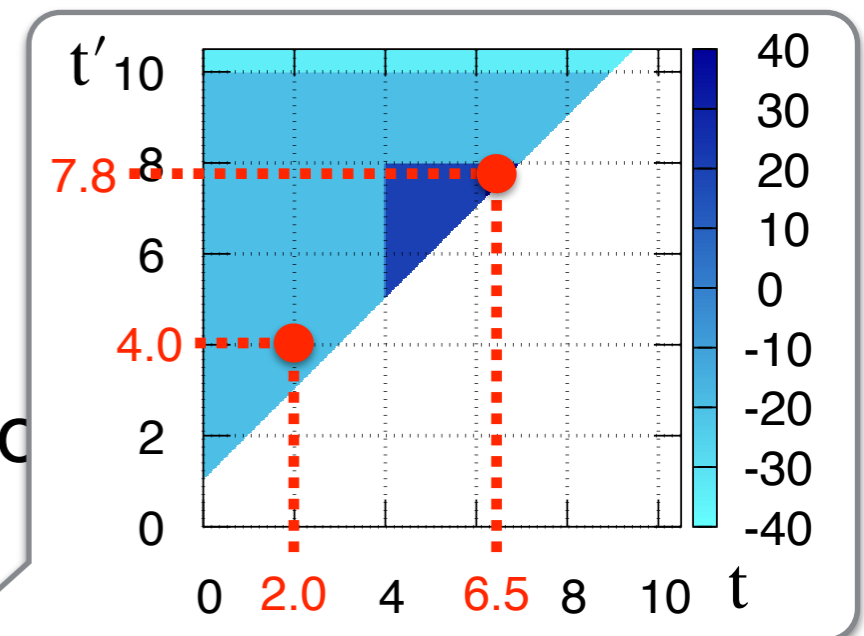
- e.g.,



- **Pattern Specification  $\mathcal{W}$**

- Spec. to be monitored

- e.g., The velocity should not keep  $> 80$  for



## Output

- **Function** assigning the satisfaction degree to each **subsignal**  $\sigma([t, t'])$

- e.g.,  $\mathcal{M}(\sigma, \mathcal{W})(2.0, 4.0) = -20$ ,  $\mathcal{M}(\sigma, \mathcal{W})(6.5, 7.8) = 40$ , ...

satisfaction degree of  $\mathcal{W}$  for  $\sigma([2.0, 4.0])$



# Online Quantitative Timed Pattern Matching with Semiring- Valued Weighted Automata

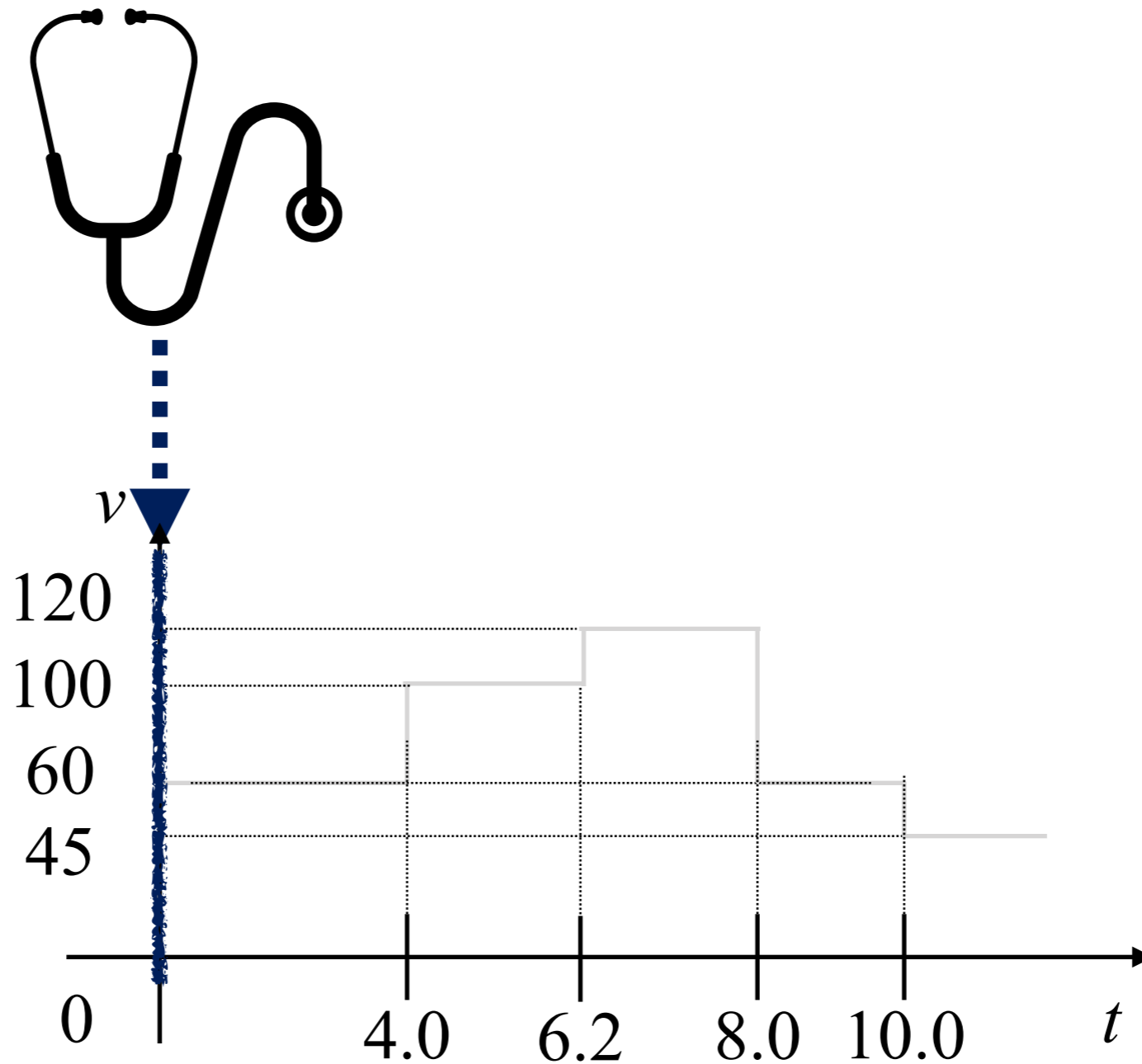
Masaki Waga

Kyoto University

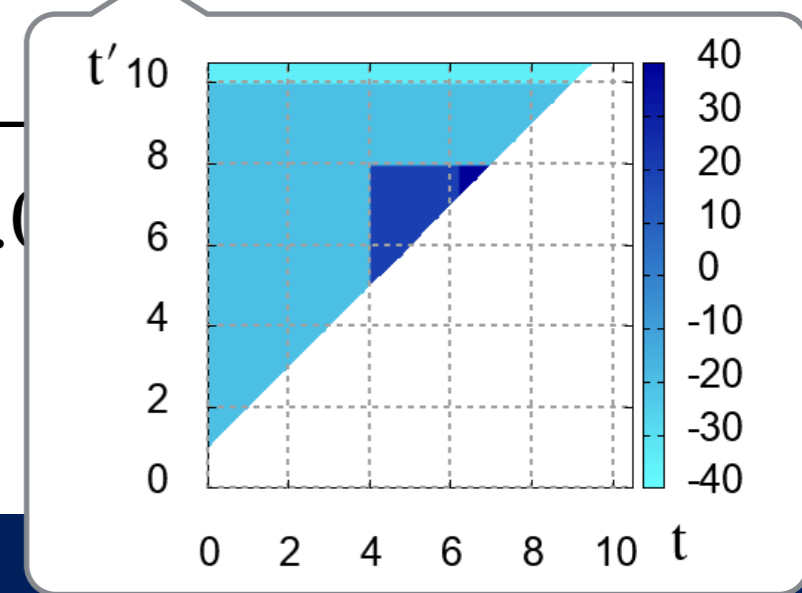
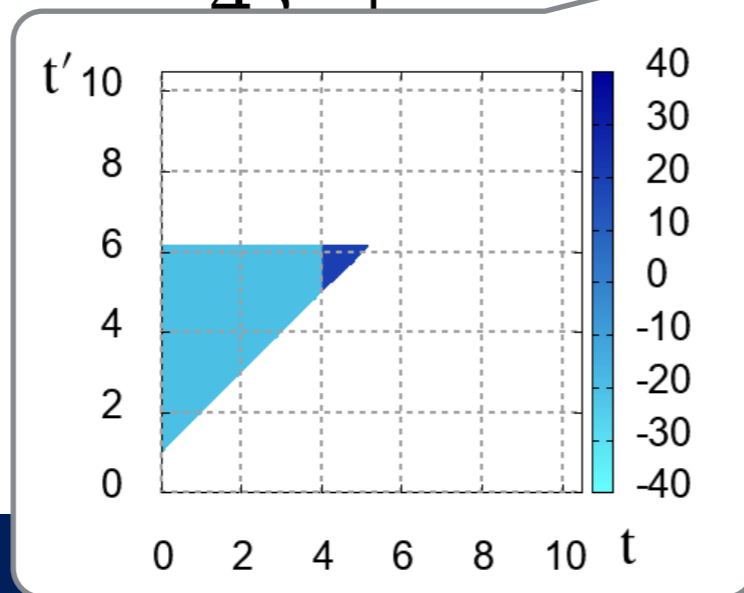
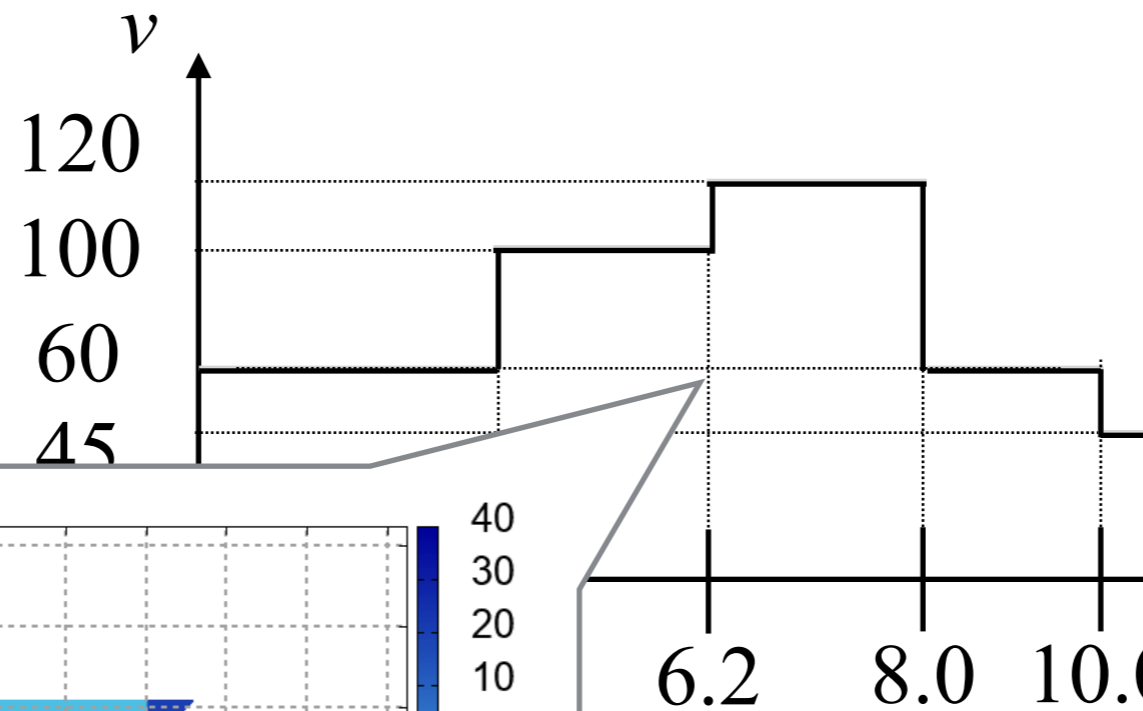
12 May 2021, YR-OWLS

Based on the paper at FORMATS'19

# Online Pattern Matching



# Online Pattern Matching





# Online Quantitative Timed Pattern Matching with Semiring- Valued Weighted Automata

Masaki Waga

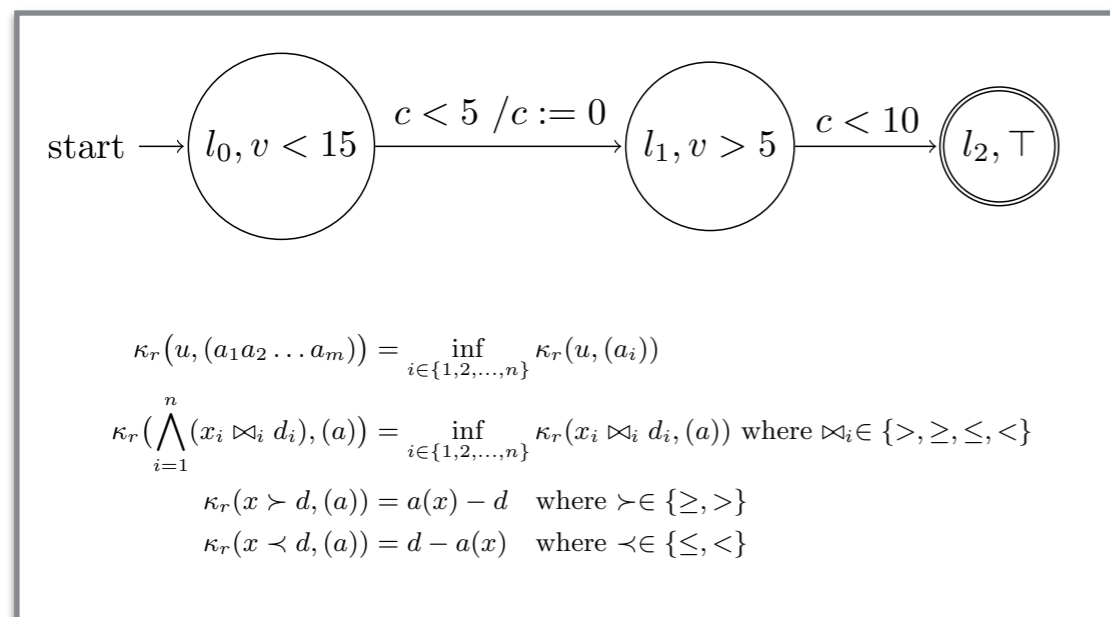
Kyoto University

12 May 2021, YR-OWLS

Based on the paper at FORMATS'19

# Timed symbolic weighted automata (TSWA)

- New formalism for spec.
- Automata structure is good for online monitoring
- Generality of semiring (same as the usual WFA)



	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	sup	inf
$\otimes$	$\wedge$	inf	+

# Contribution

- Introduced timed symbolic weighted automata (TSWA)
- **TSWA**: timed automata with signal constraints (TSA)
  - **Automata structure**
  - + semiring-valued weight function
  - **Quantitative semantics**
- Gave online algorithm for quantitative timed pattern matching
- Implementation + experiments → **Scalable!!**

# Related Works

	Qualitative	Quantitative
Offline	[Ulus+, FORMATS'14] (TRE)	[Bakhirkin+, FORMATS'17] (Signal RE)
Online	[Ulus+, TACAS'16], [Bakhirkin+, FORMATS'18] (TRE & TA)	<b>[Contribution]</b> <b>(TSWA)</b>

Timed automata



Timed automata  
with  
**signal constraints**

**Only “Robust” Semantics**  
[Fainekos & Pappas, TCS'09]

**Any Semantics defined by semiring-valued weight function**

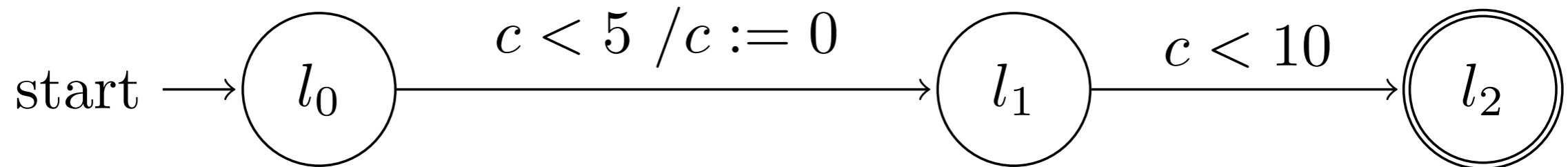


# Outline

- Motivation + Introduction
- Technical Part
  - Timed symbolic weighted automata (TSWA)
    - TSWA: TA with signal constraints + weight function
  - Quantitative monitoring/timed pattern matching algorithm
    - Idea: zone construction with weight
- Experiments

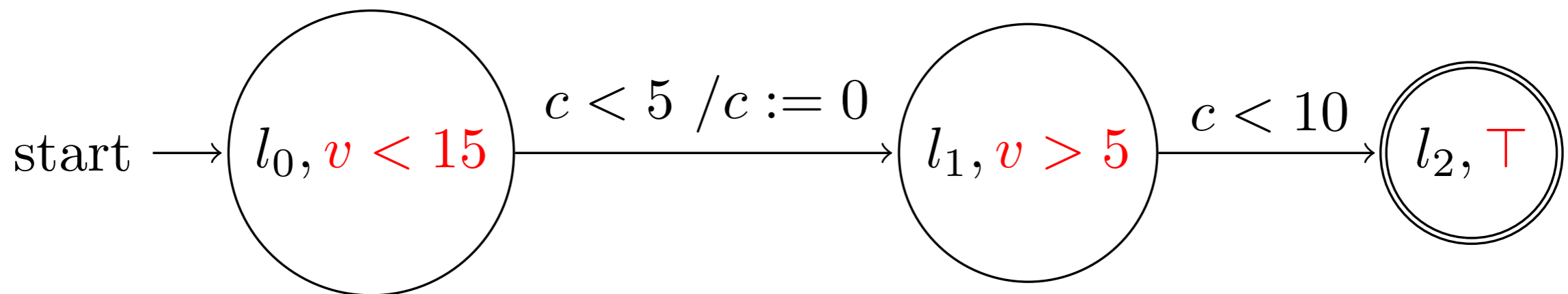
# TSWA: TA with signal constraints + weight function

## Timed Automaton (TA)



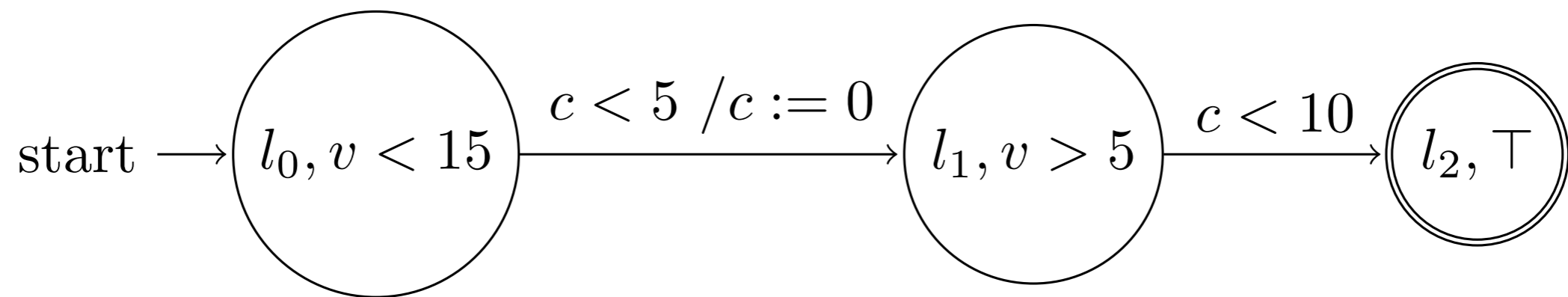
# TSWA: TA with signal constraints + weight function

## Timed Symbolic Automaton (TSA)



# TSWA: TA with signal constraints + weight function

## Timed Symbolic Weighted Automaton (TSWA)



+

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, m\}} \kappa_r(u, (a_i))$$

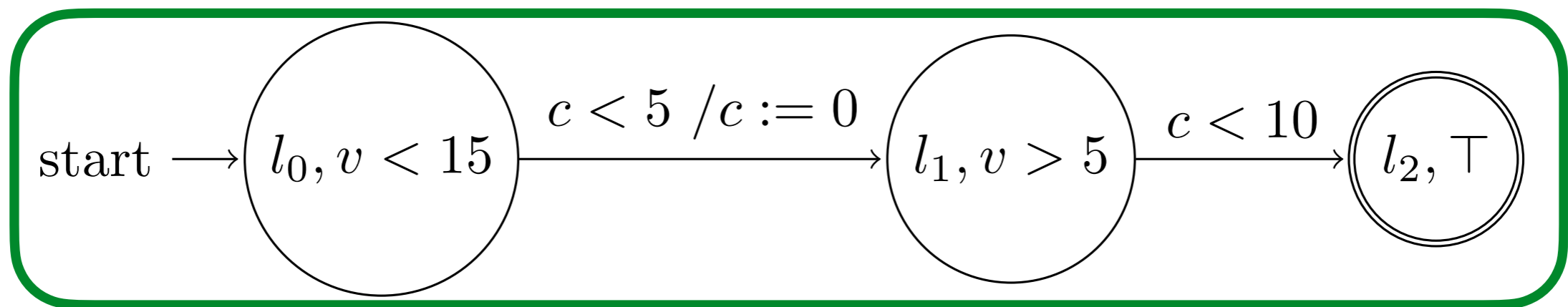
$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \text{ where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \text{ where } \prec \in \{\leq, <\}$$

# TSWA: TA with signal constraints + weight function

## Timed Symbolic Weighted Automaton (TSWA)



**Automata structure**

+

**Quantitative semantics**

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, m\}} \kappa_r(u, (a_i))$$

$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \quad \text{where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \quad \text{where } \prec \in \{\leq, <\}$$

# Weight function

$$\kappa: \Phi(X, \mathbb{D}) \times (\mathbb{D}^X)^{\otimes} \rightarrow S$$

Constraints on signal values at the location

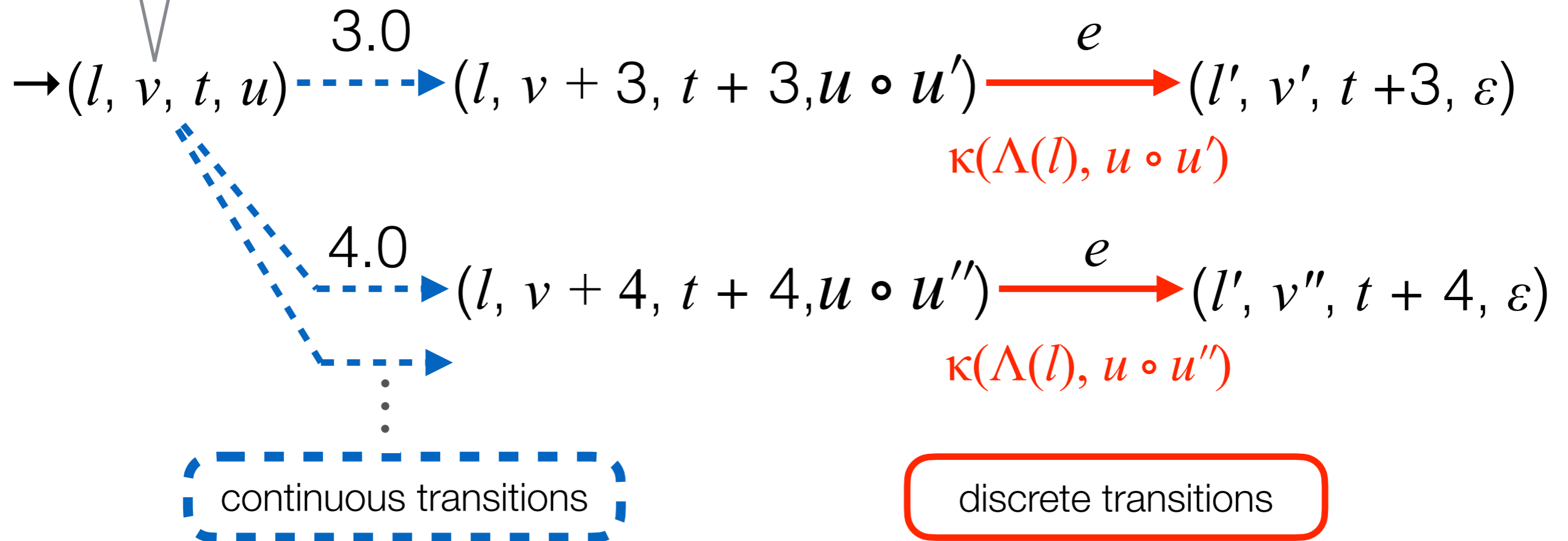
Sequence of signal values at the location

Semiring value

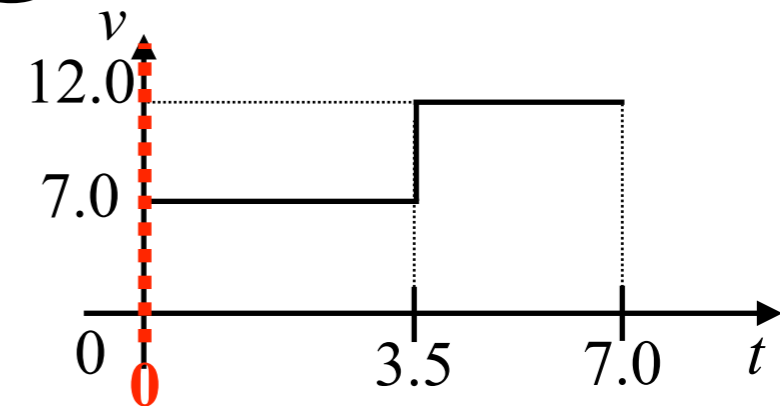
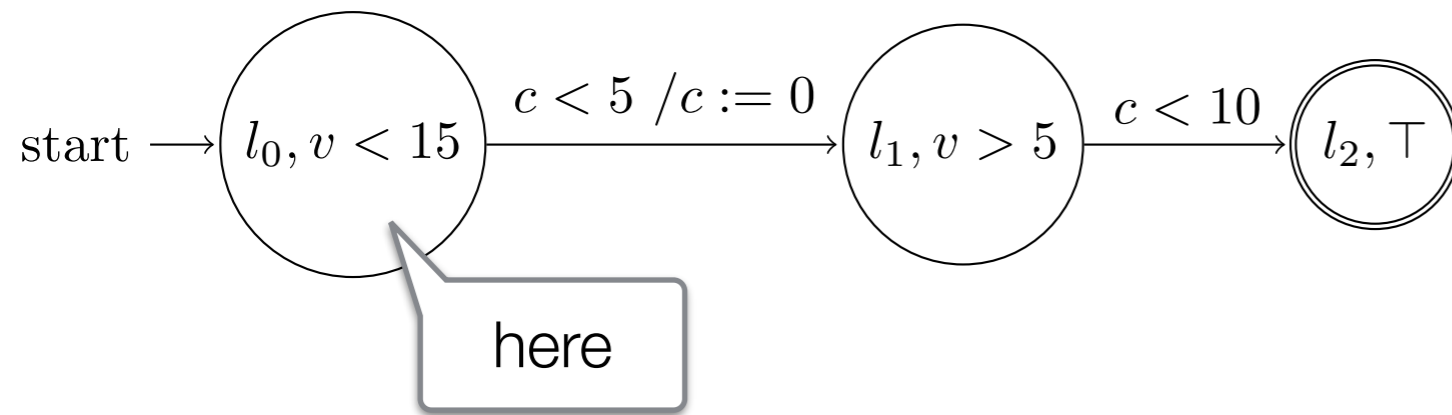
- $\kappa(\Lambda(l), u)$ : weight for the stay at  $l$  with signal values  $u$
- Semiring: set  $S$  with accumulating operators  $\oplus$  and  $\otimes$
- We can use any complete and idempotent semiring

# Semantics: Weighted TTS

- $l$ : location
- $v$ : clock valuation
- $t$ : absolute time
- $u$ : sequence of signal values after the latest discrete transition



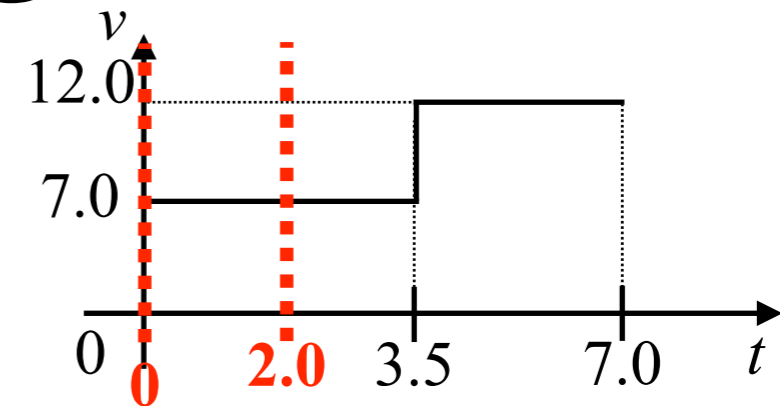
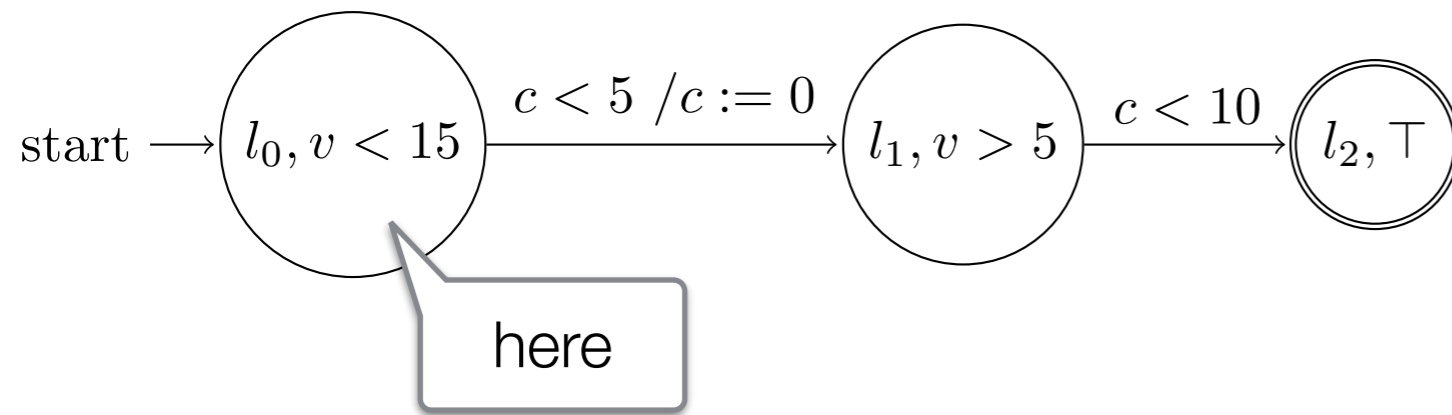
# One path in Weighted TTS



$\rightarrow (l_0, c=0, 0, \varepsilon)$

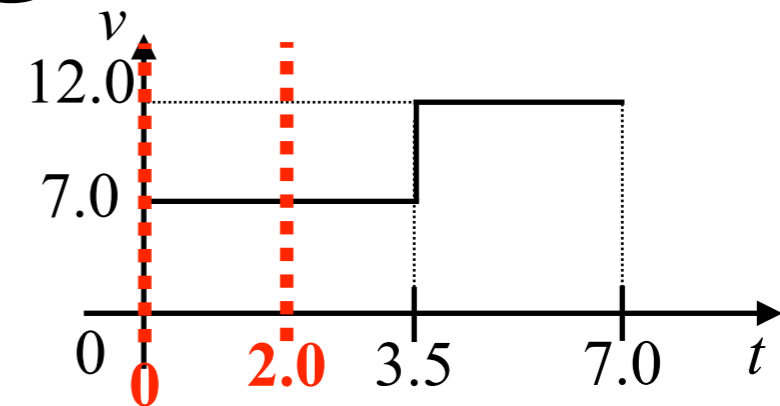
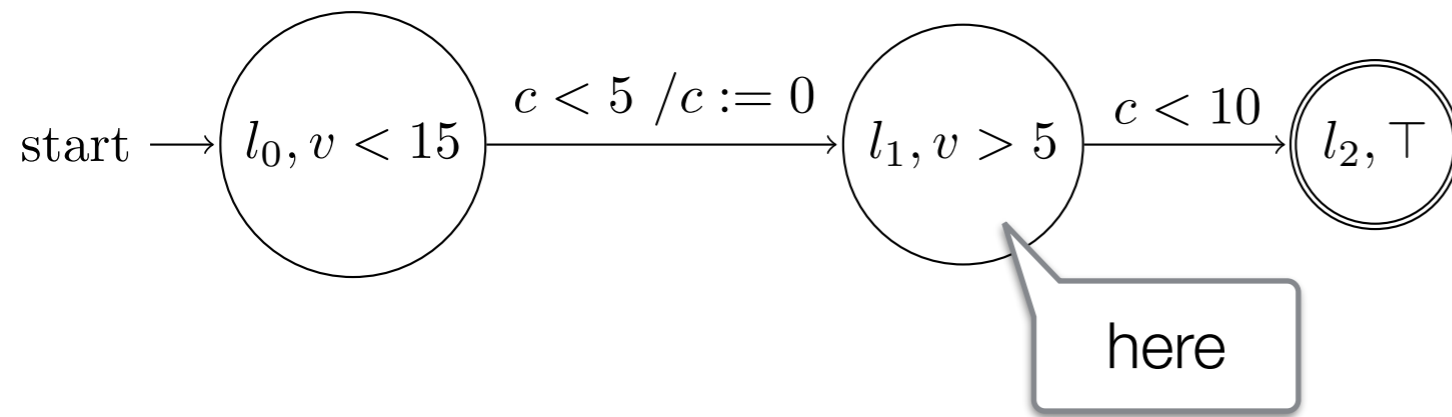


# One path in Weighted TTS



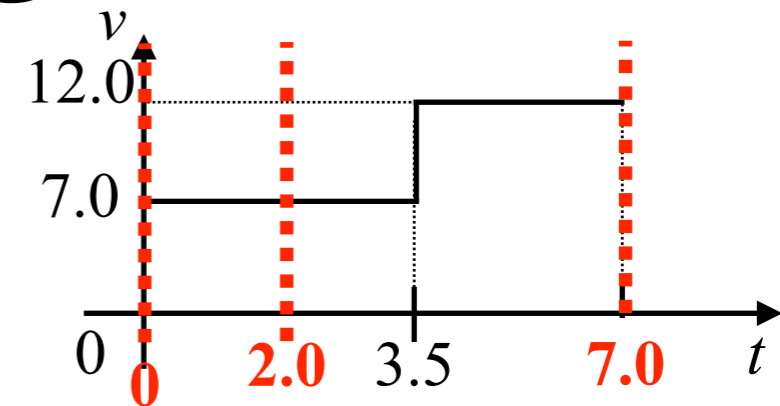
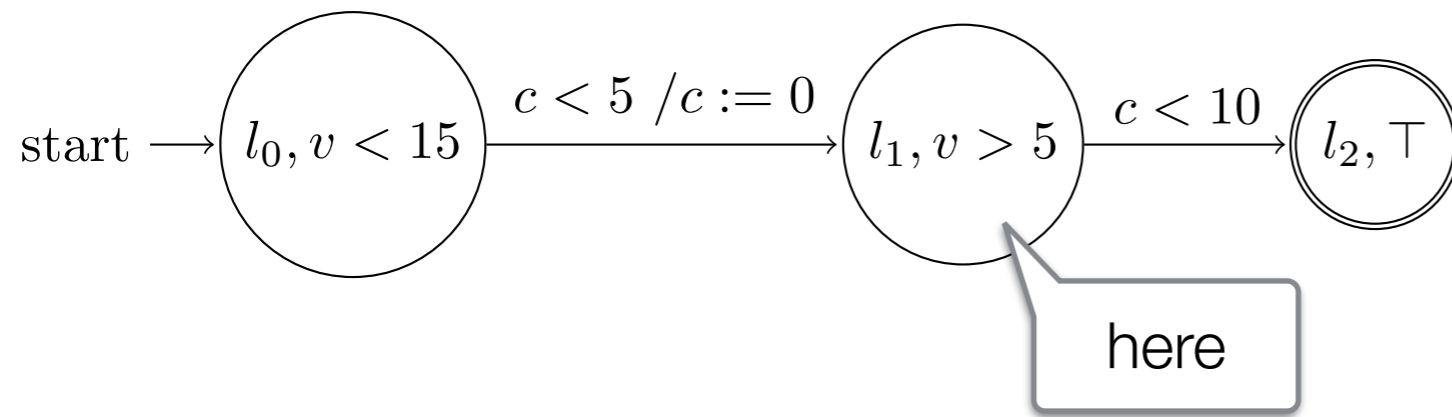
$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v = 7\})$$

# One path in Weighted TTS



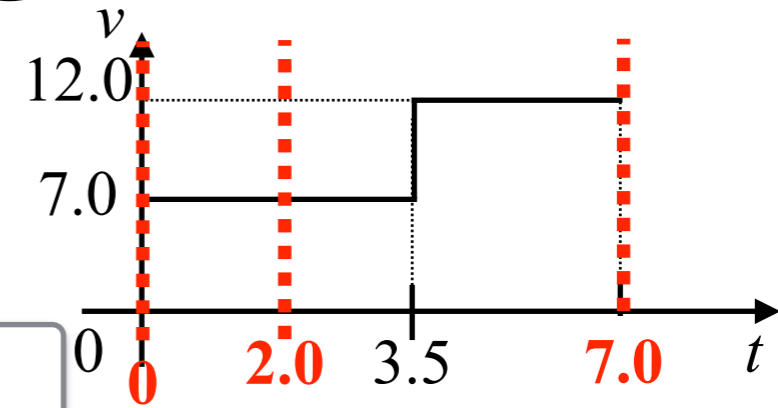
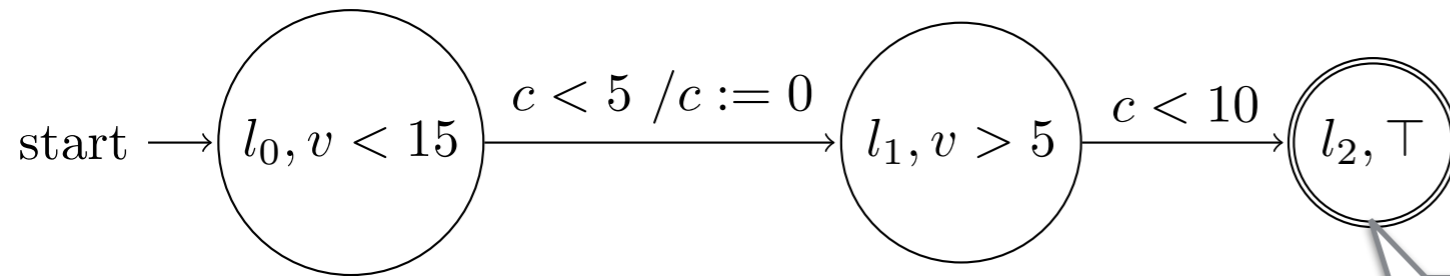
$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v = 7\}) \xrightarrow[\kappa(v < 15, \{v=7\})]{e} (l_1, c=0, 2, \varepsilon)$$

# One path in Weighted TTS



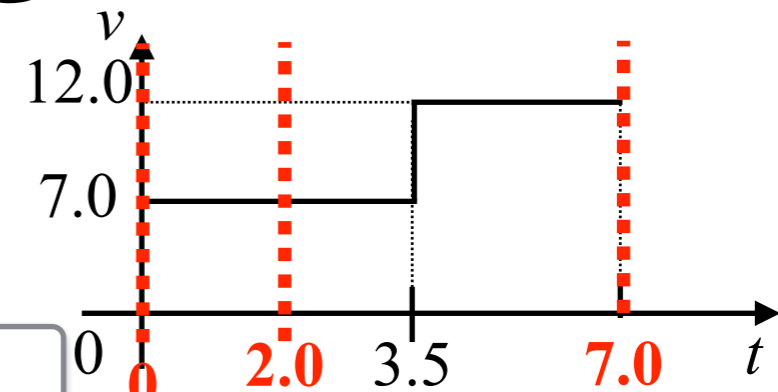
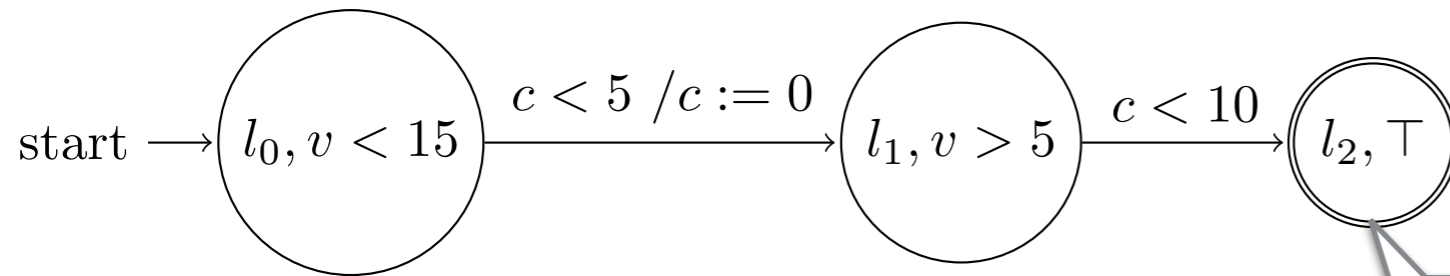
$$\begin{aligned}
 &\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v=7\}) \xrightarrow[\kappa(v < 15, \{v=7\})]{e} (l_1, c=0, 2, \varepsilon) \\
 &\quad \xrightarrow{5.0} (l_1, c=5, 7, \{v=7\}\{v=12\})
 \end{aligned}$$

# One path in Weighted TTS



$$\begin{aligned}
 &\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v=7\}) \xrightarrow[\kappa(v < 15, \{v=7\})]{e} (l_1, c=0, 2, \varepsilon) \\
 &\quad \xrightarrow{5.0} (l_1, c=5, 7, \{v=7\}\{v=12\}) \xrightarrow[\kappa(v > 5, \{v=7\}\{v=12\})]{e} (l_2, c=5, 7, \varepsilon)
 \end{aligned}$$

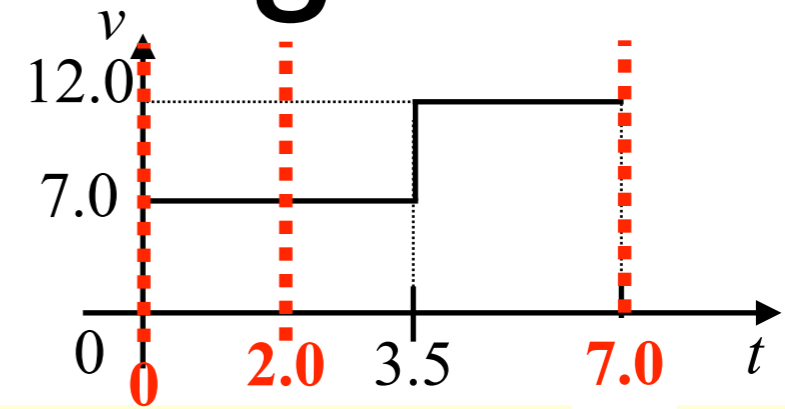
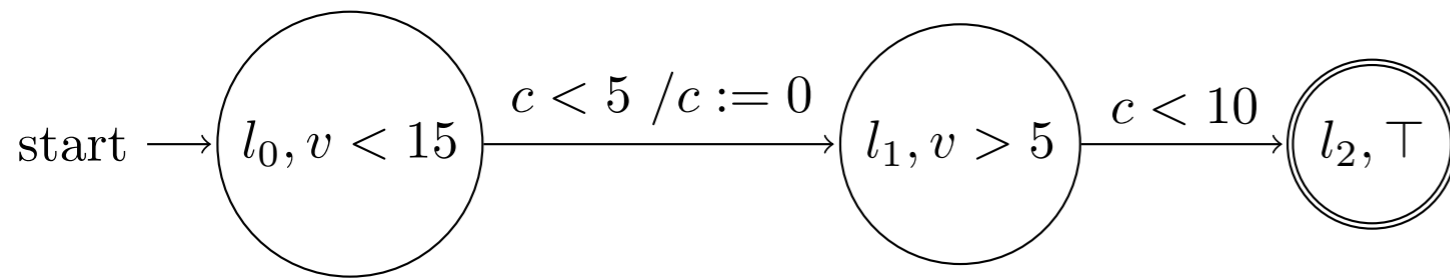
# One path in Weighted TTS



$$\begin{aligned}
 &\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v=7\}) \xrightarrow[\kappa(v < 15, \{v=7\})]{e} (l_1, c=0, 2, \varepsilon) \\
 &\quad \xrightarrow{5.0} (l_1, c=5, 7, \{v=7\} \{v=12\}) \xrightarrow[\kappa(v > 5, \{v=7\} \{v=12\})]{\otimes e} (l_2, c=5, 7, \varepsilon)
 \end{aligned}$$

	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	sup	inf
$\otimes$	$\wedge$	inf	+

# Accumulating paths in Weighted TTS



$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v=7\}) \xrightarrow{e} (l_1, c=0, 3, \varepsilon) \xrightarrow{5.0} (l_1, c=5, 7, \{v=7\}\{v=12\}) \xrightarrow{e} (l_2, c=5, 7, \varepsilon)$$

$$\kappa(v < 15, \{v=7\}) \otimes \kappa(v > 5, \{v=7\}\{v=12\})$$

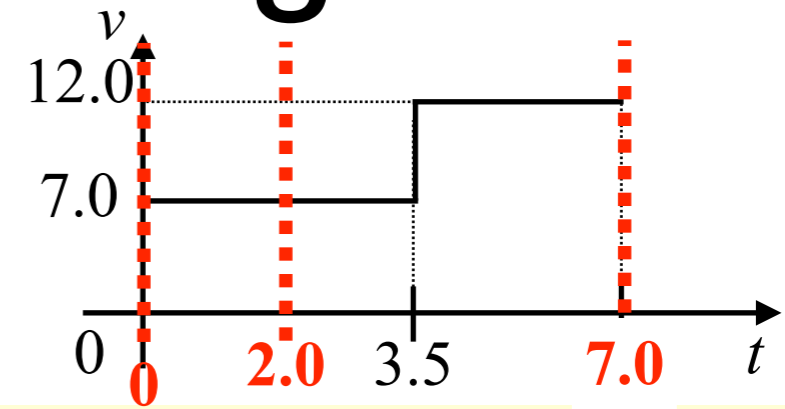
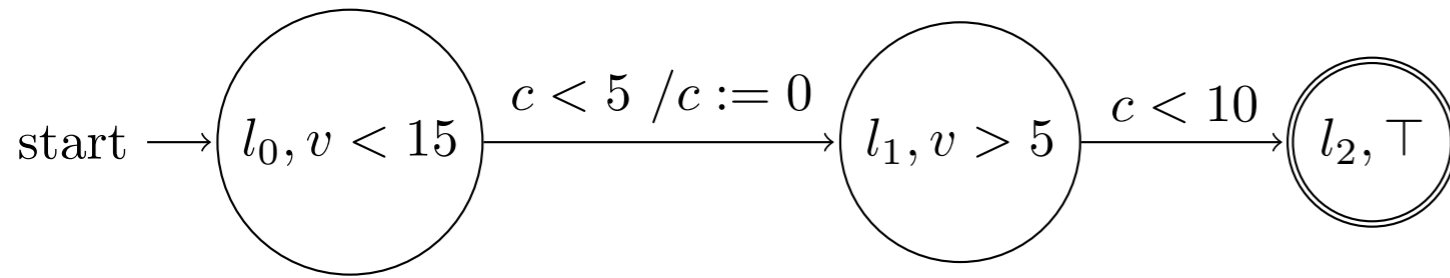
$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{4.0} (l_0, c=4, 4, \{v=7\}\{v=12\}) \xrightarrow{e} (l_1, c=0, 4, \varepsilon) \xrightarrow{3.0} (l_1, c=3, 7, \{v=12\}) \xrightarrow{e} (l_2, c=3, 7, \varepsilon)$$

$$\kappa(v < 15, \{v=7\}\{v=12\}) \otimes \kappa(v > 5, \{v=12\})$$

⋮

	Boolean	sup-inf	tropical
$\mathcal{S}$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	sup	inf
$\otimes$	$\wedge$	inf	+

# Accumulating paths in Weighted TTS



$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v=7\}) \xrightarrow{e} (l_1, c=0, 3, \varepsilon) \xrightarrow{5.0} (l_1, c=5, 7, \{v=7\}\{v=12\}) \xrightarrow{e} (l_2, c=5, 7, \varepsilon)$$

$$\kappa(v < 15, \{v=7\}) \otimes \kappa(v > 5, \{v=7\}\{v=12\})$$



$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{4.0} (l_0, c=4, 4, \{v=7\}\{v=12\}) \xrightarrow{e} (l_1, c=0, 4, \varepsilon) \xrightarrow{3.0} (l_1, c=3, 7, \{v=12\}) \xrightarrow{e} (l_2, c=3, 7, \varepsilon)$$

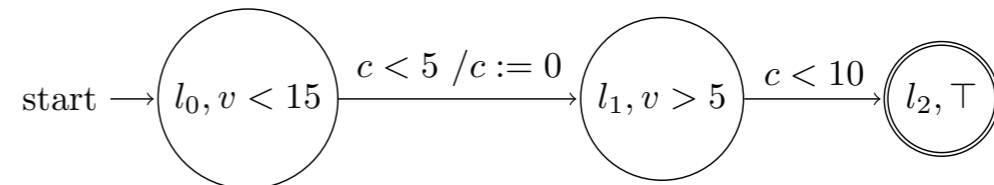
$$\kappa(v < 15, \{v=7\}\{v=12\}) \otimes \kappa(v > 5, \{v=12\})$$



	Boolean	sup-inf	tropical
$\mathcal{S}$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	sup	inf
$\otimes$	$\wedge$	inf	+

# Timed symbolic weighted automata (TSWA)

**TSA**: the automata structure



**Weight function** ( $\kappa$ ): the one-step semantics  
(weight on each transition)

**Semiring operations** ( $\otimes, \oplus$ ): how to accumulate  
weights

One-step semantics  $\rightarrow$  semantics for a path/TSWA



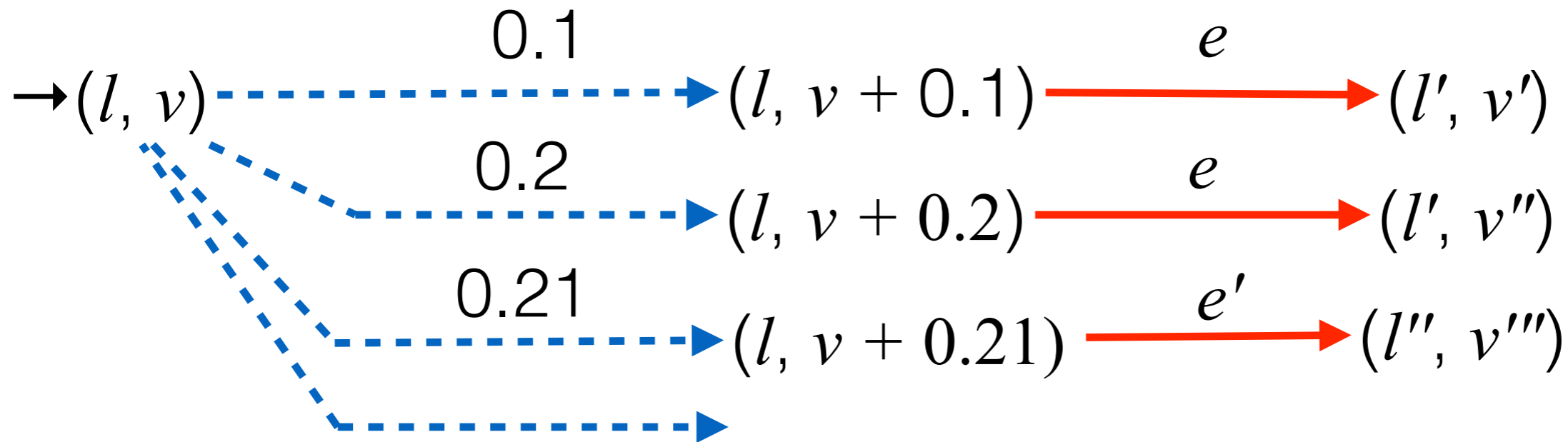
# Outline

- Motivation + Introduction
- Technical Part
  - Timed symbolic weighted automata (TSWA)
    - TSWA: TA with signal constraints + weight function
  - Quantitative monitoring/timed pattern matching algorithm
    - Idea: zone construction with weight
- Experiments

# Review: Reachability by zones

continuous transitions

discrete transitions



⋮

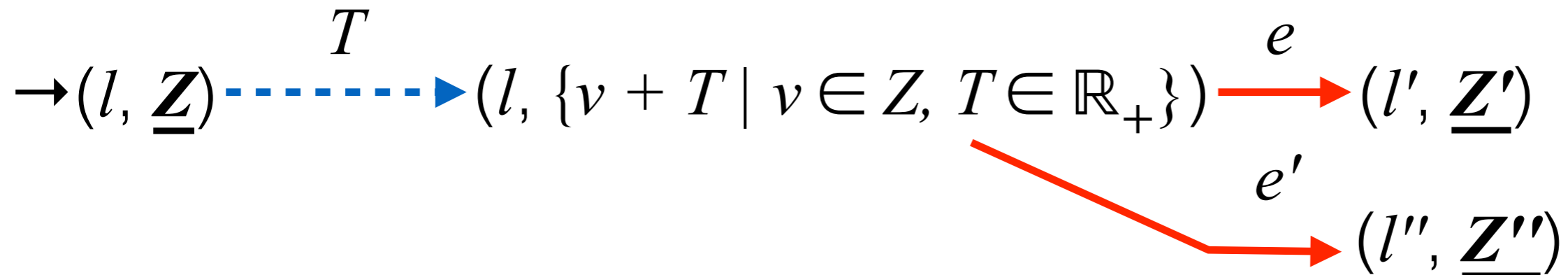
**Infinitely** many delays!!

**Infinitely** many reachable states!! → symbolic analysis by **zones**

# Review: Reachability by zones

continuous transitions

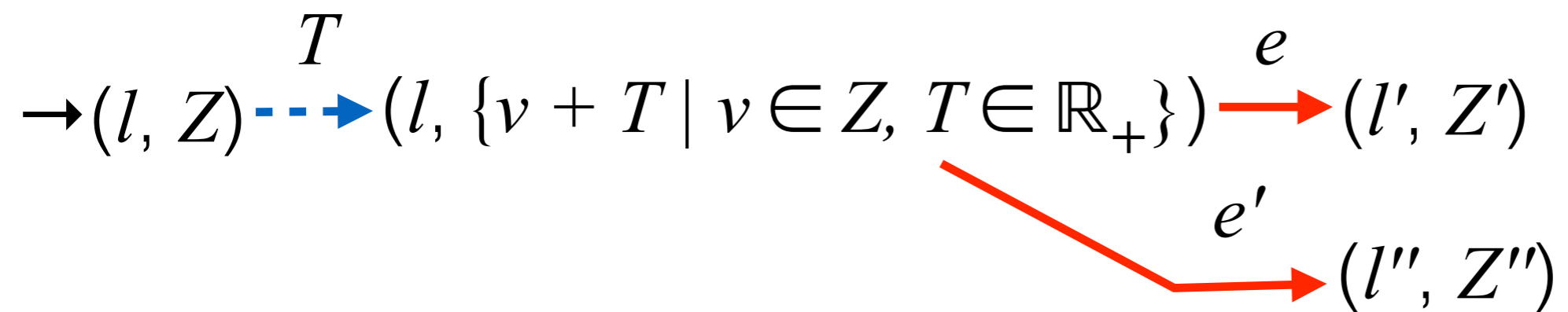
discrete transitions



**Infinitely** many reachable states!!  $\rightarrow$  symbolic analysis by **zones**

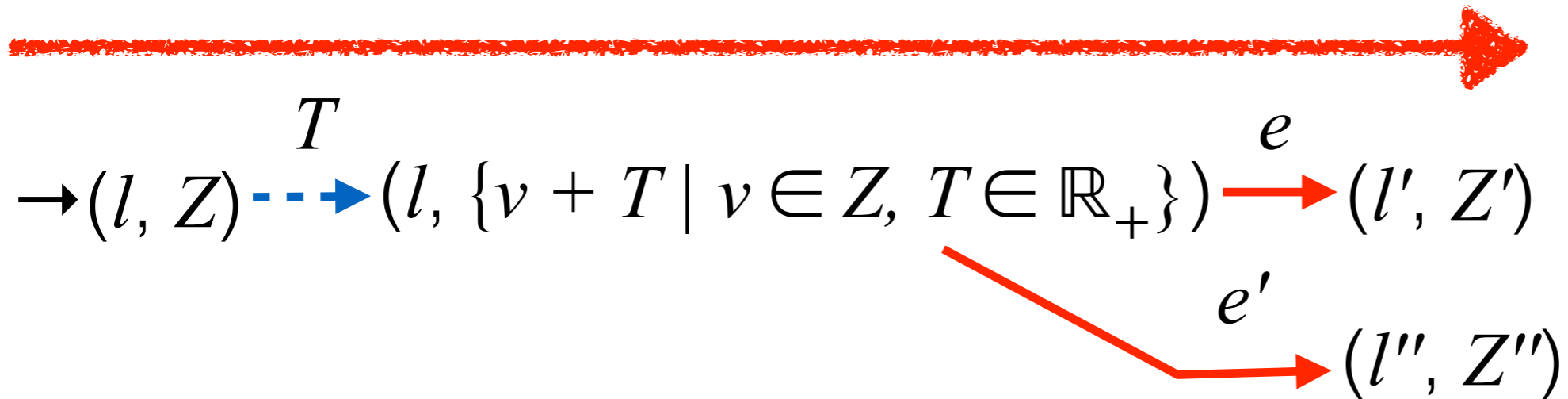
**Observation: reachability  
is shortest distance over  
Boolean semiring!**

# Observation: reachability is shortest distance over Boolean semiring!



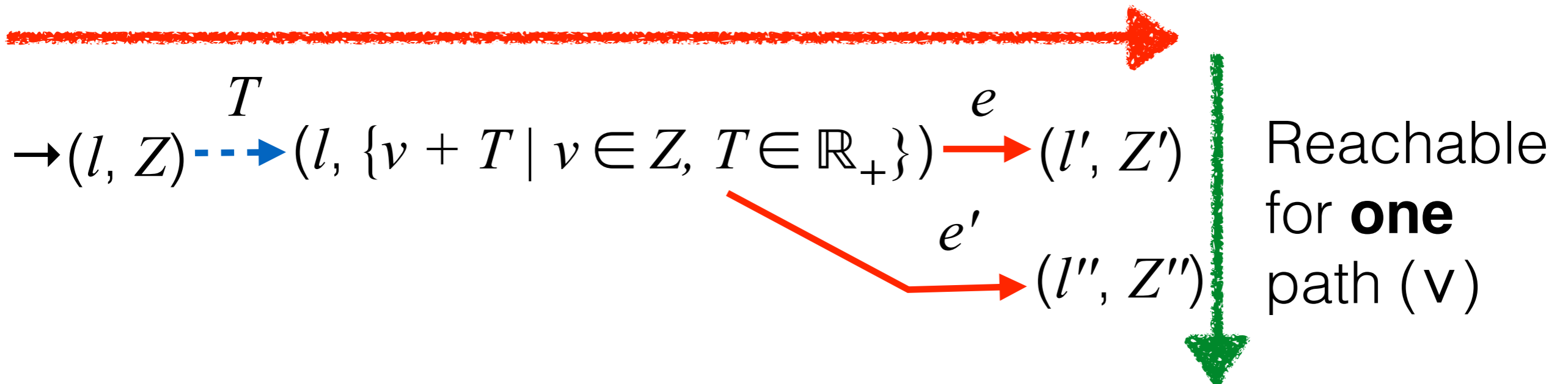
# Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions ( $\wedge$ )



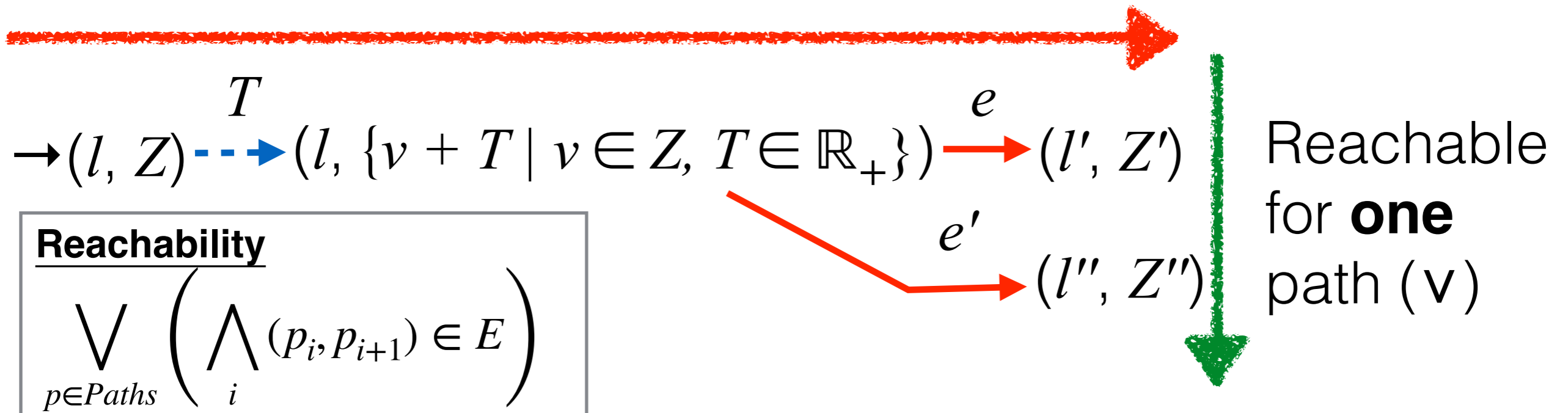
# Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions ( $\wedge$ )



# Observation: reachability is shortest distance over Boolean semiring!

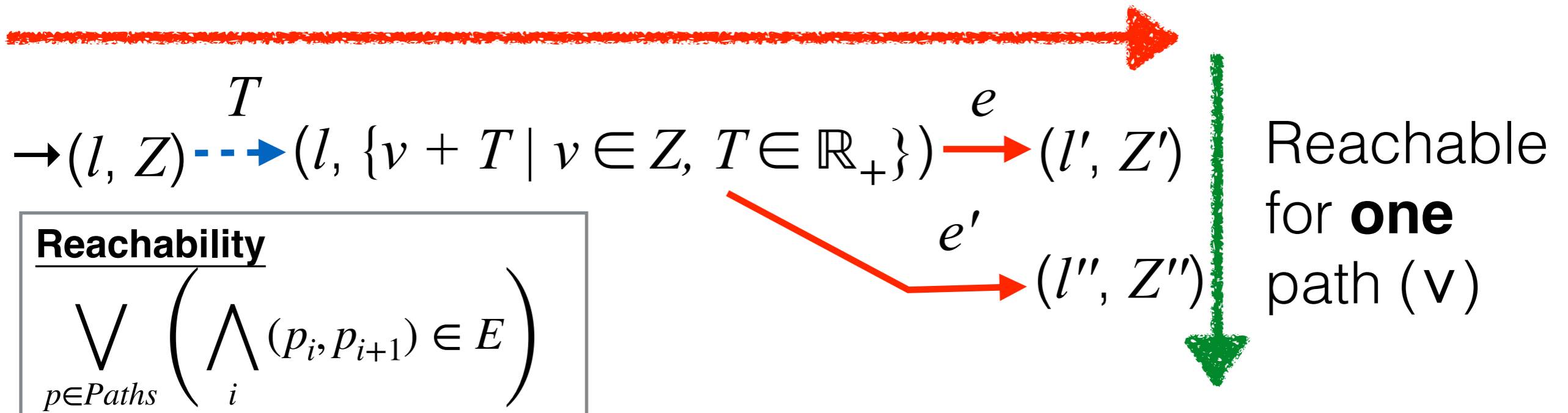
Reachable at **all** the transitions ( $\wedge$ )





# Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions ( $\wedge$ )



**Shortest Distance (for semiring)**

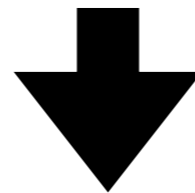
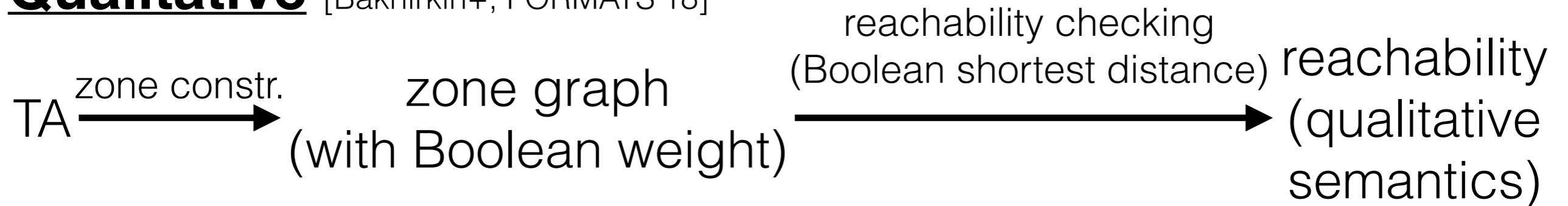
$$\bigoplus_{p \in Paths} \left( \bigotimes_i w(p_i, p_{i+1}) \right)$$

	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	sup	inf
$\otimes$	$\wedge$	inf	+

M. Waga (NII)

# Observation: reachability is shortest distance over Boolean semiring!

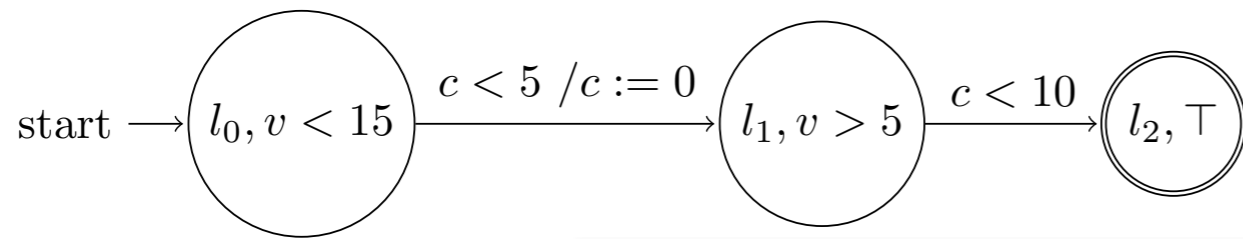
## Qualitative [Bakhirkin+, FORMATS'18]



## Quantitative [Contribution]

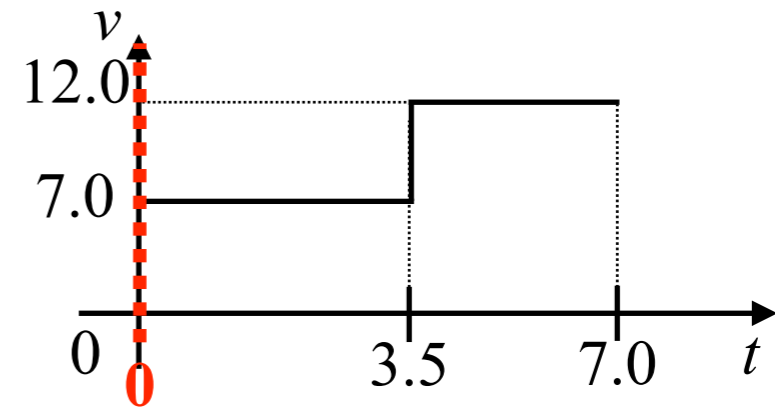


# Zone construction with weight



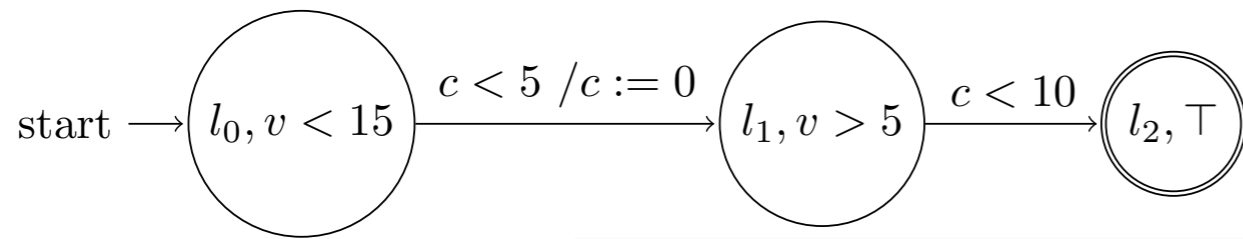
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring



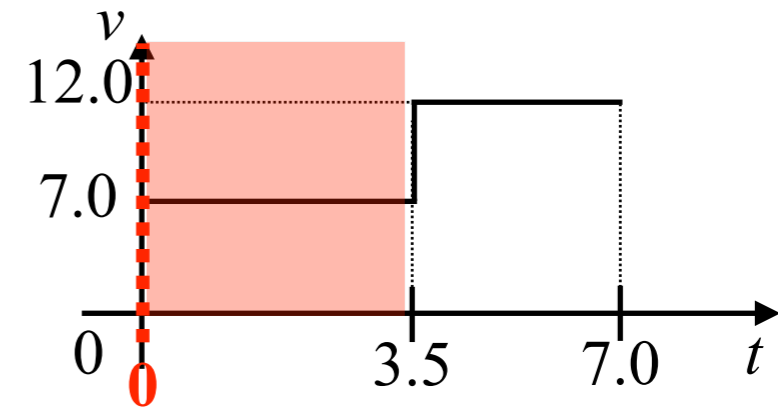
$\rightarrow (l_0, c = T = 0, \varepsilon)$

# Zone construction with weight



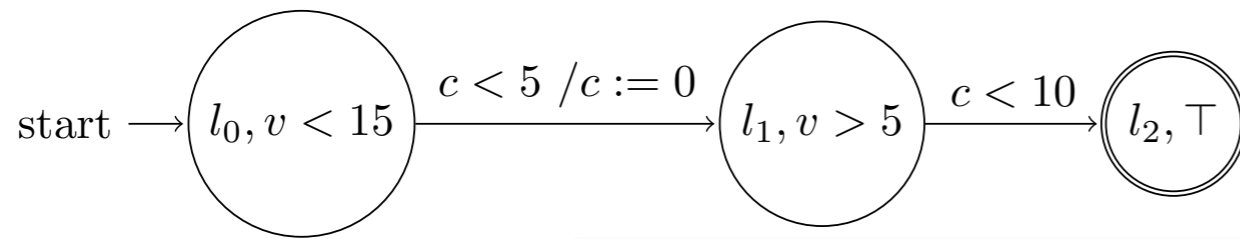
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring



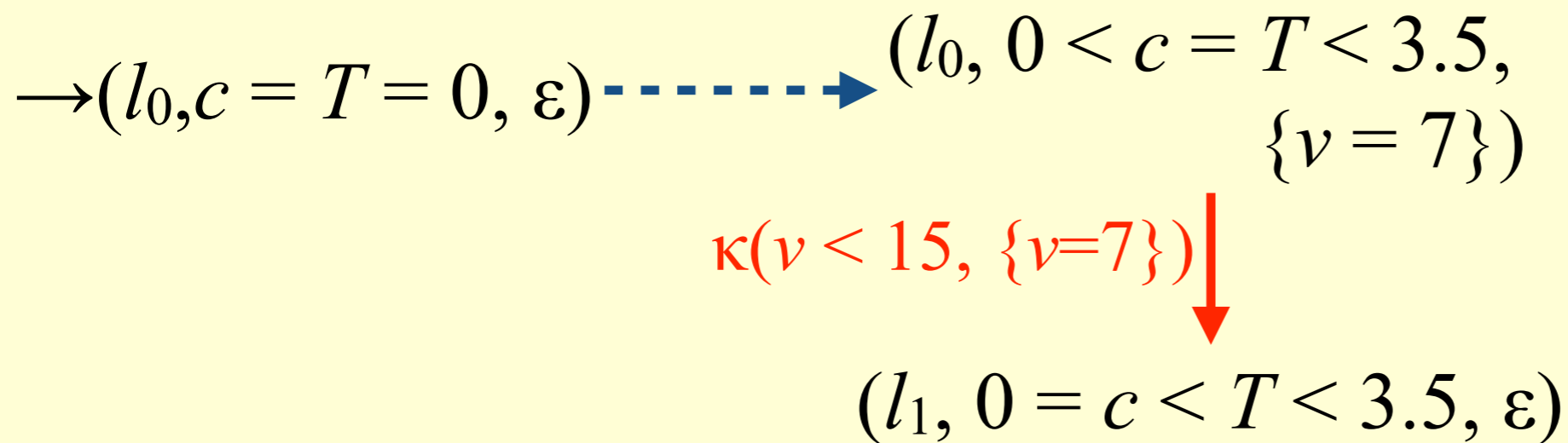
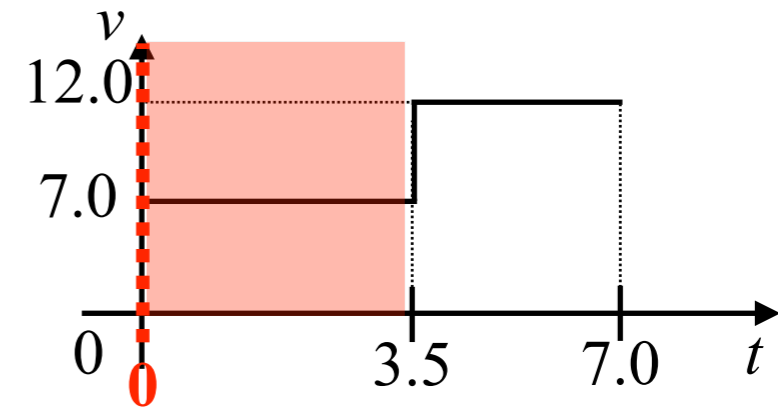
$$\rightarrow (l_0, c = T = 0, \varepsilon) \dashrightarrow (l_0, 0 < c = T < 3.5, \{v = 7\})$$

# Zone construction with weight

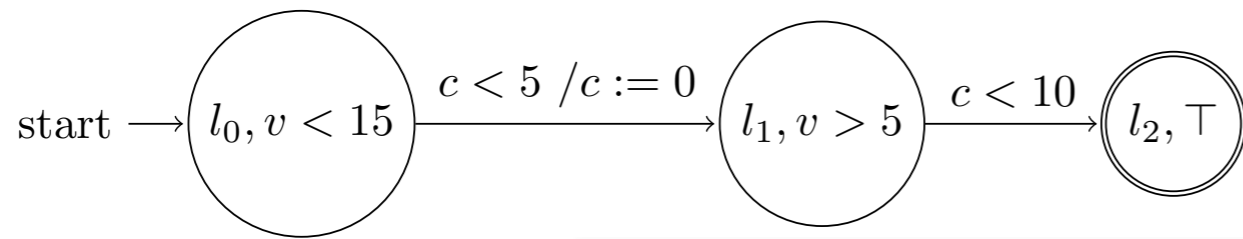


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

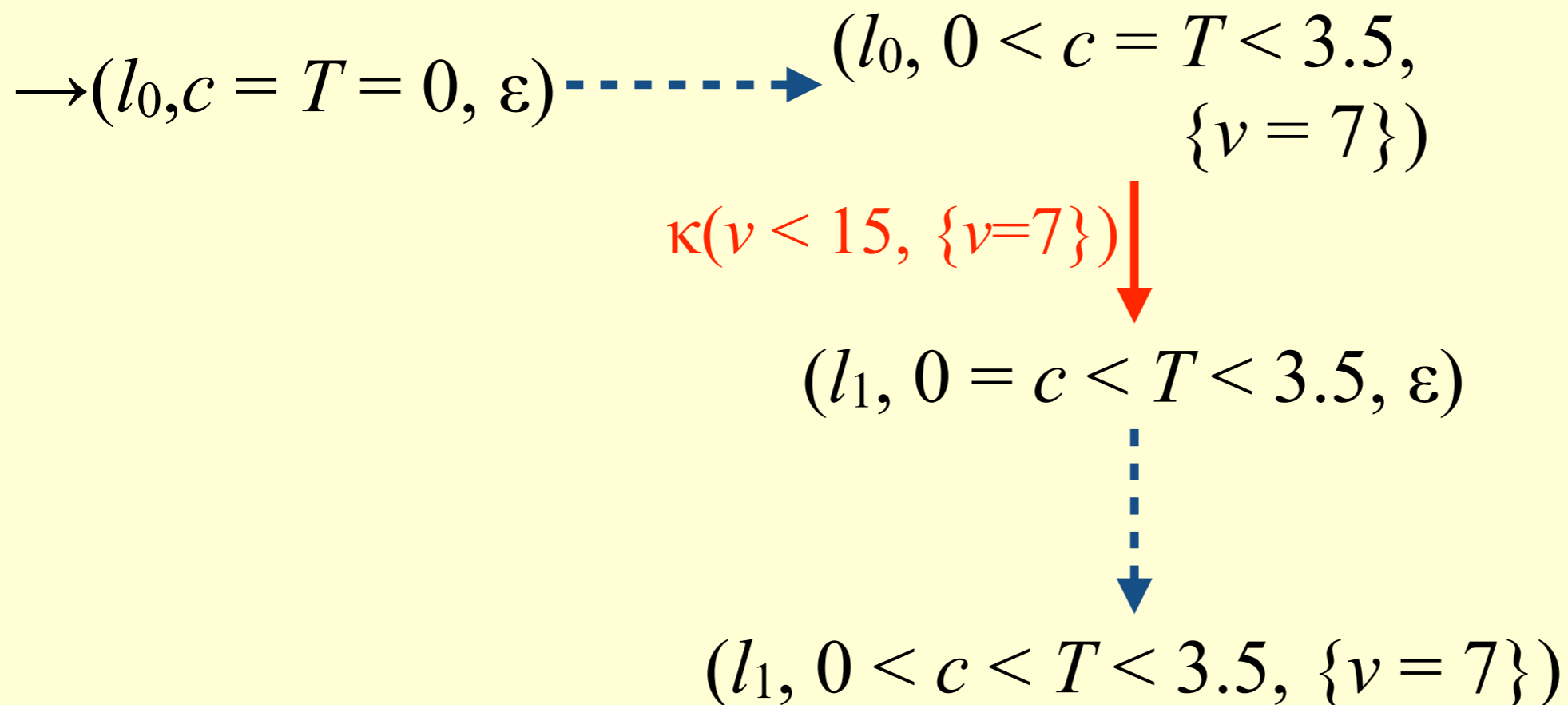
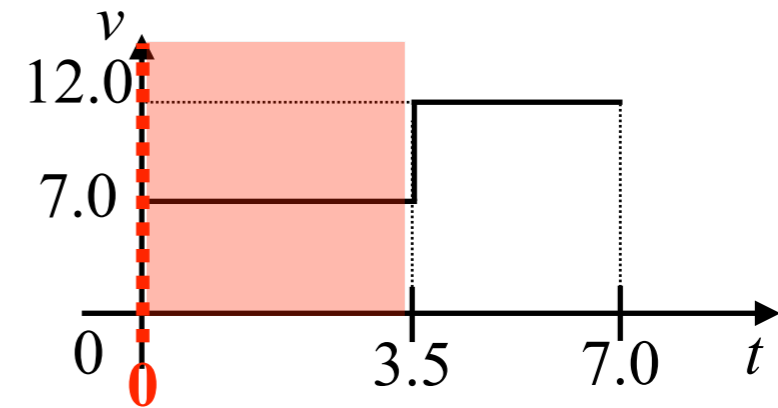


# Zone construction with weight

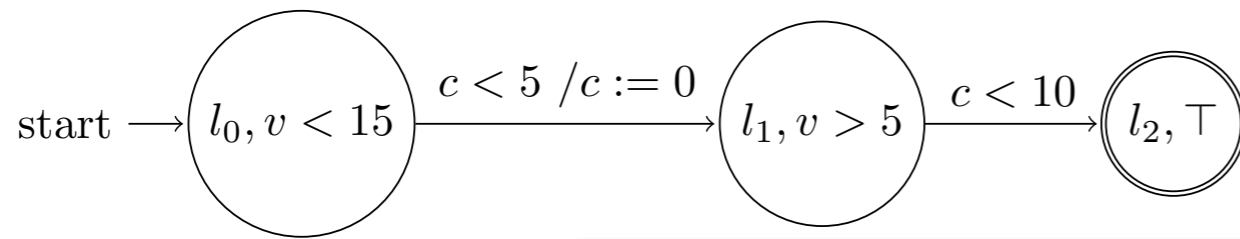


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

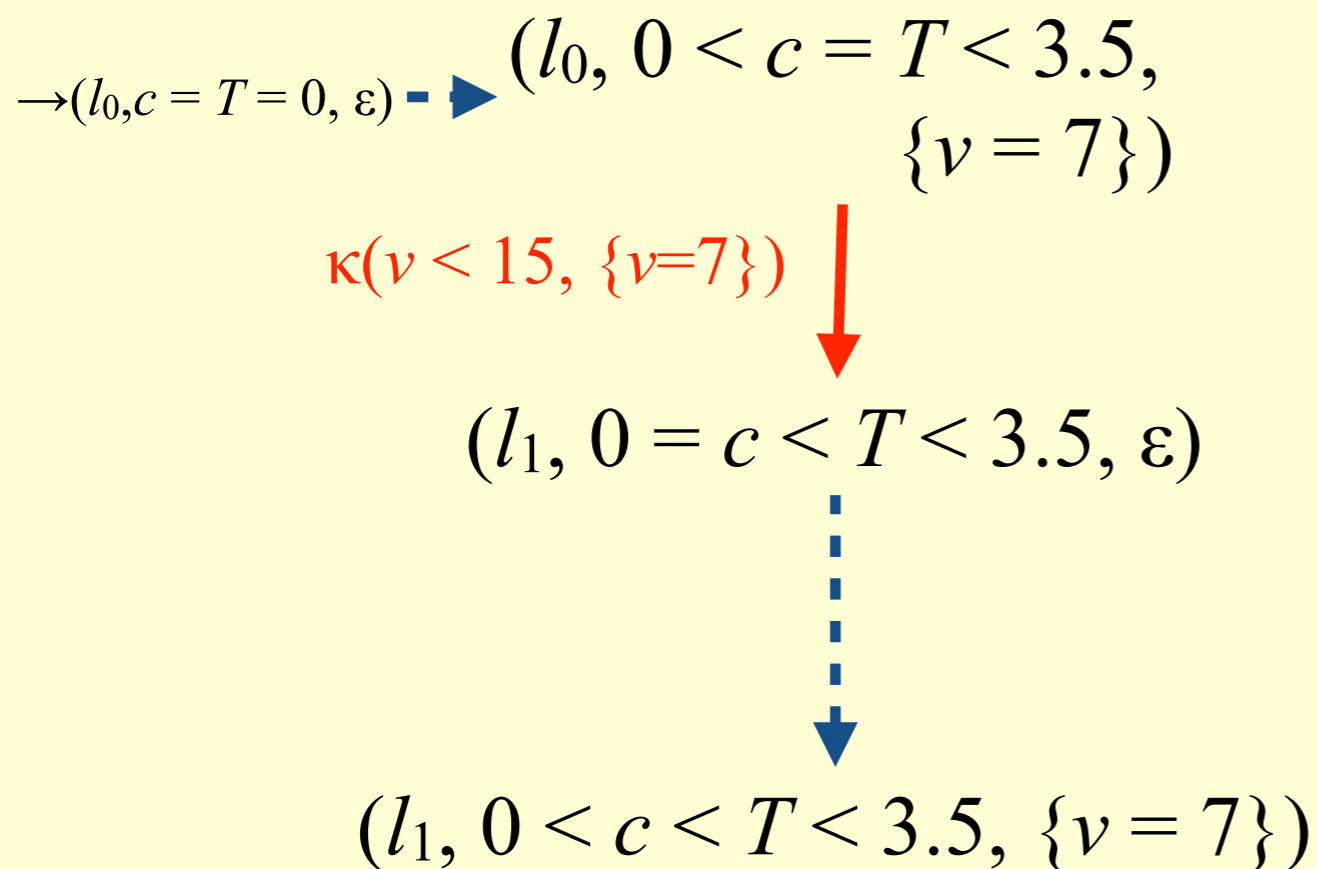
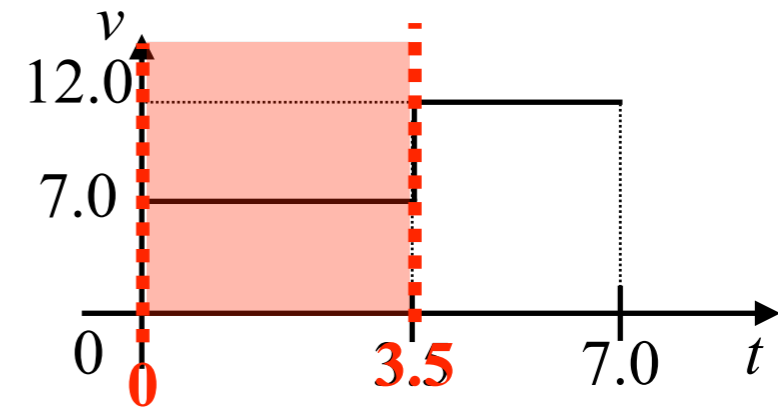


# Zone construction with weight

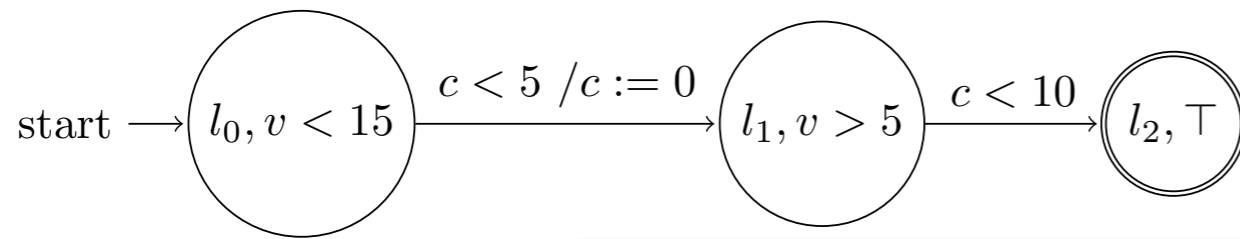


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

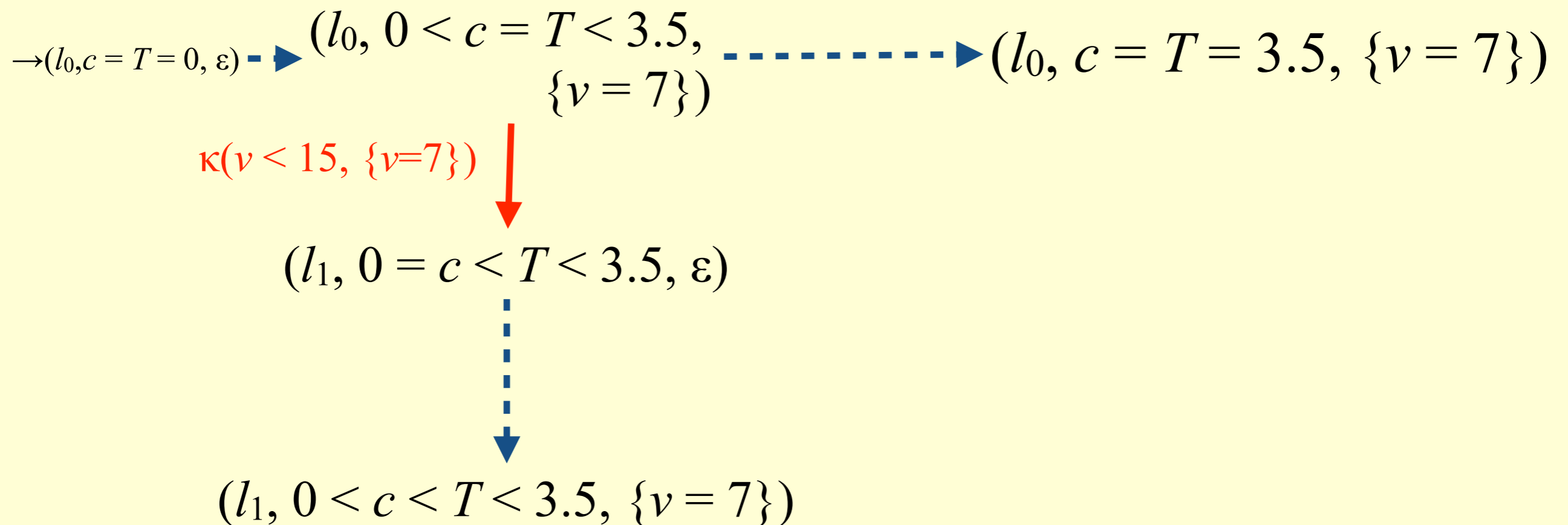
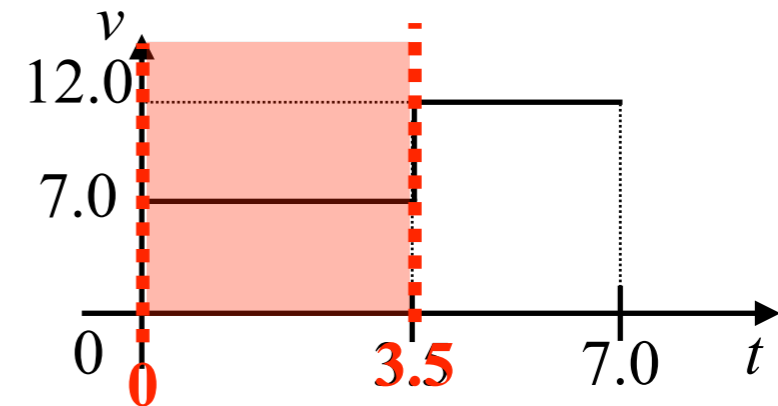


# Zone construction with weight



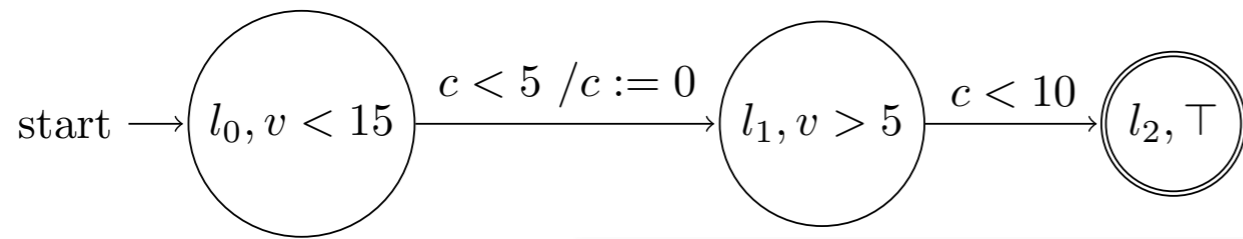
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring



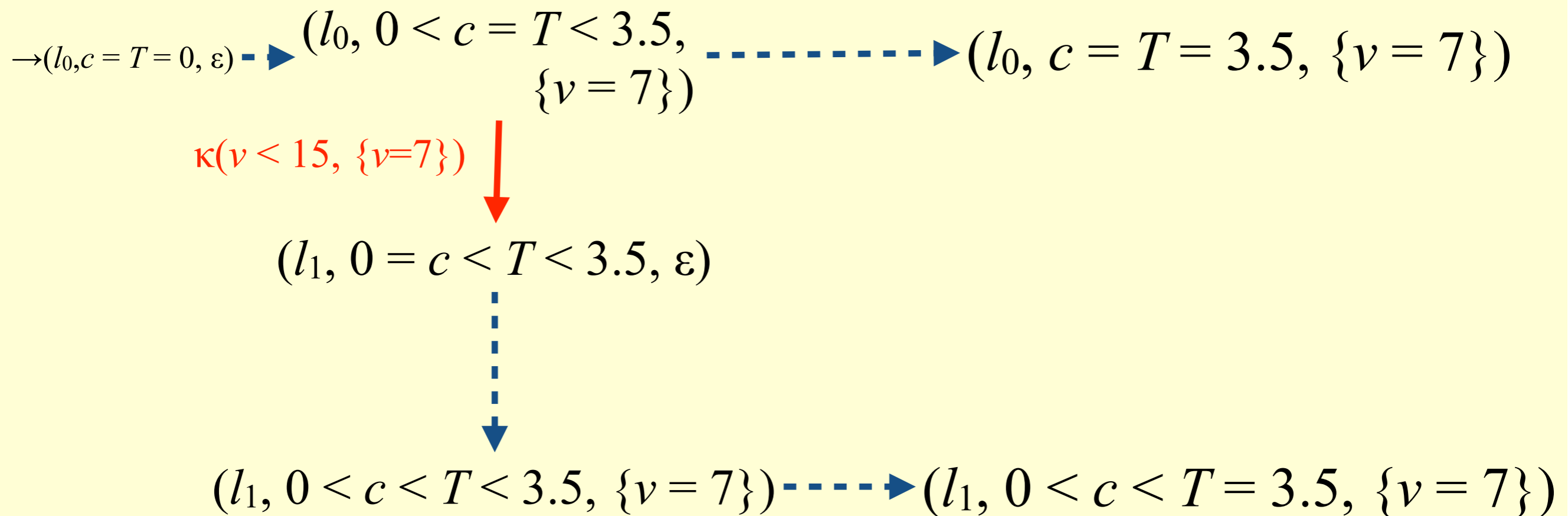
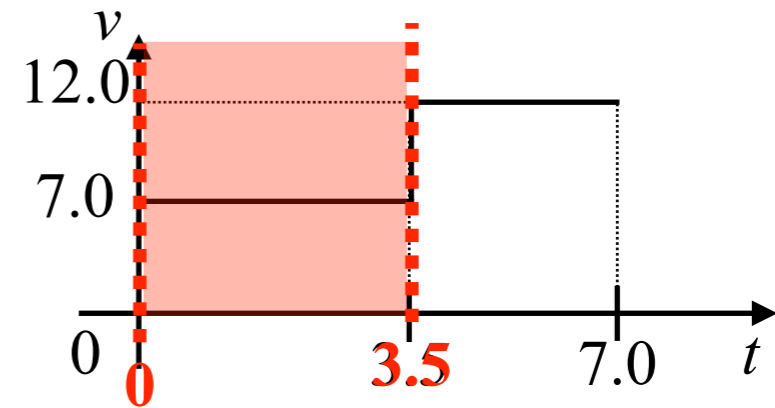


# Zone construction with weight

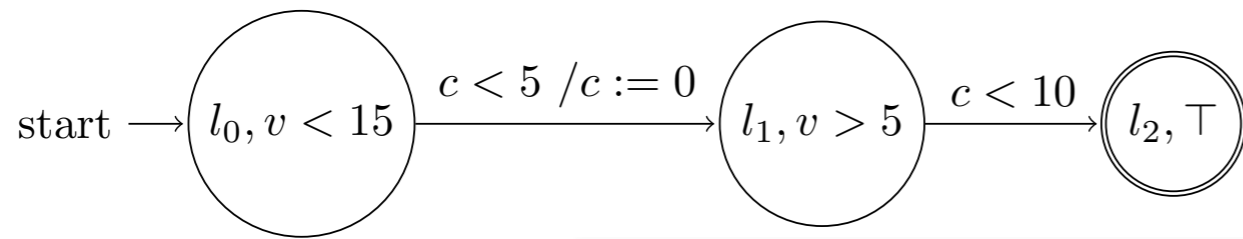


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

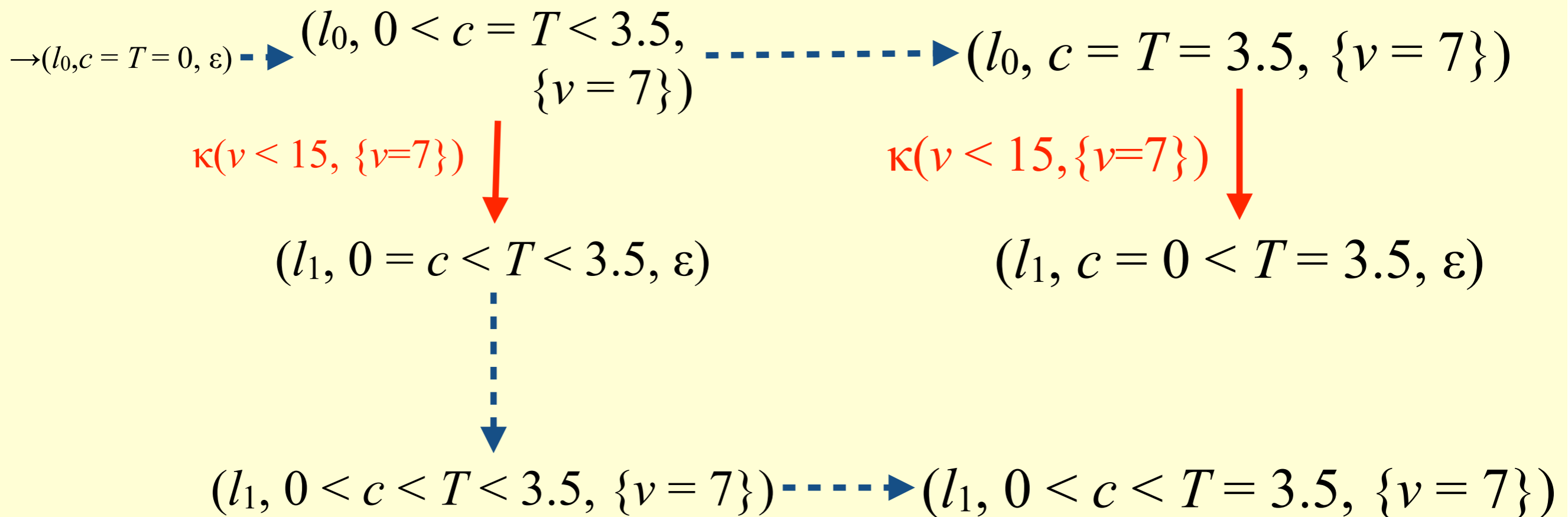
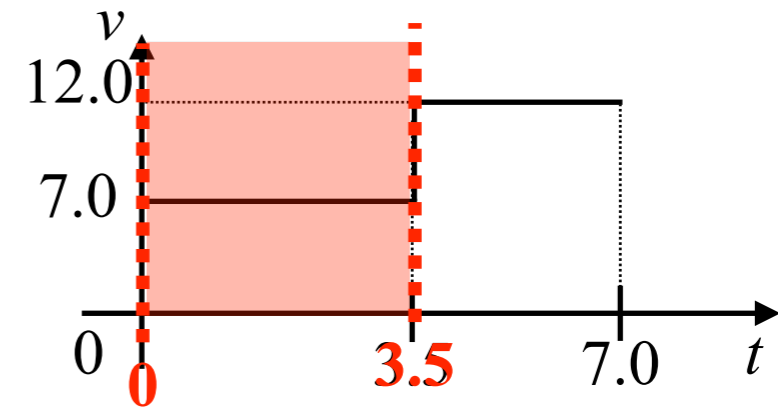


# Zone construction with weight

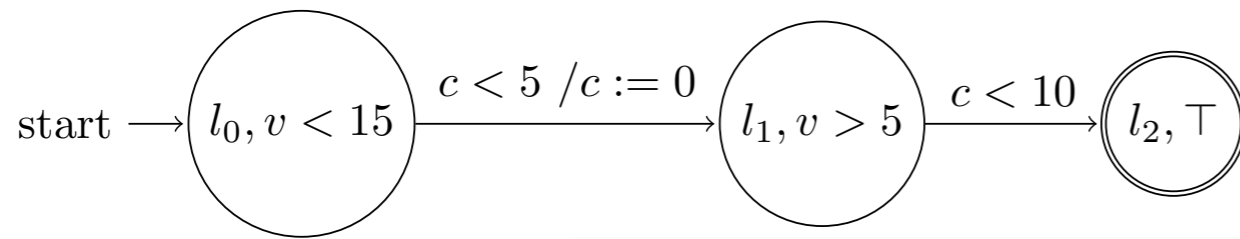


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

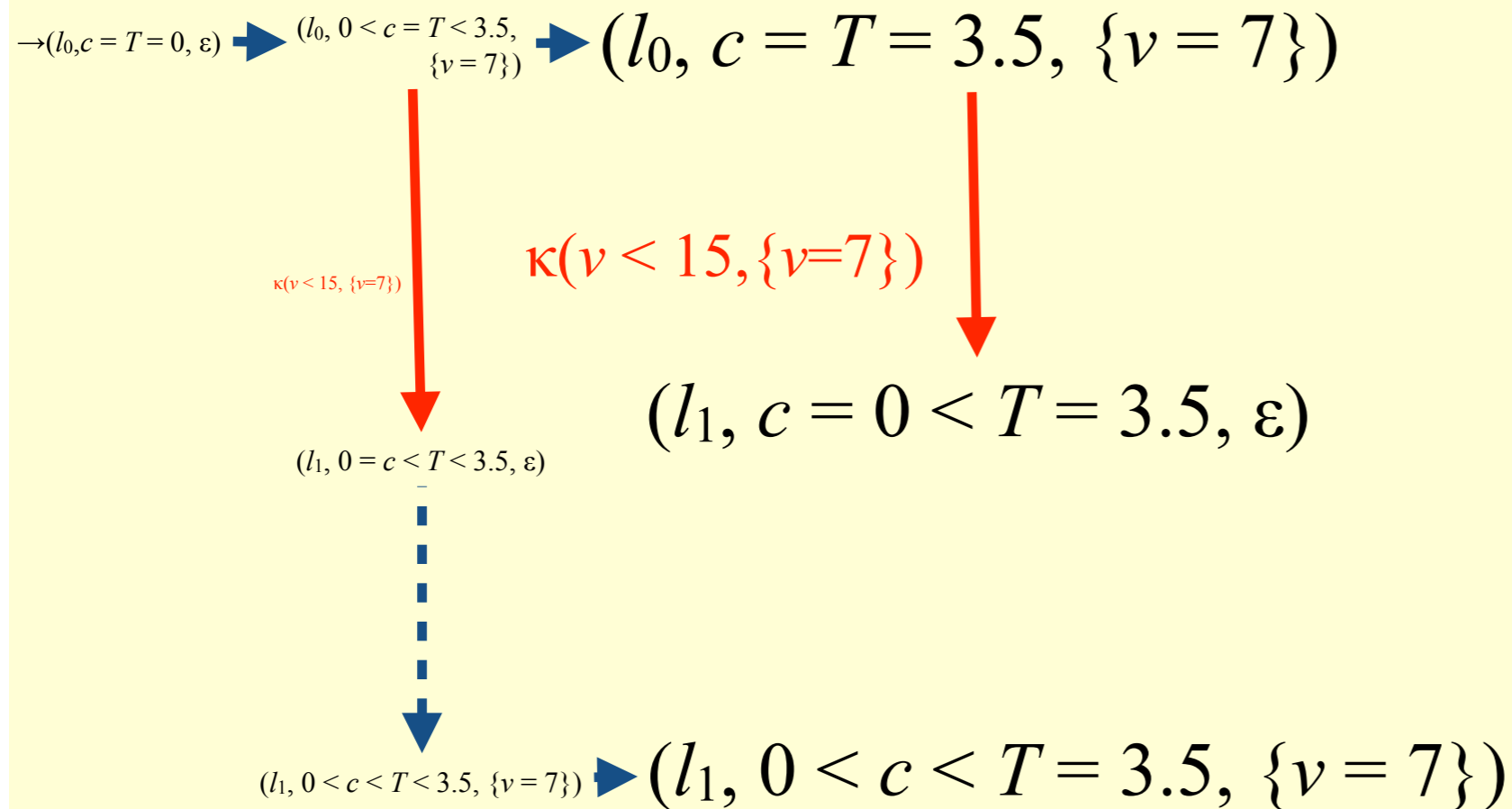
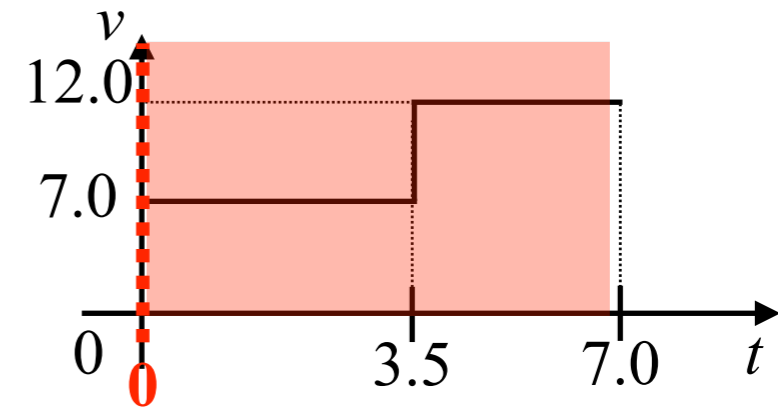


# Zone construction with weight

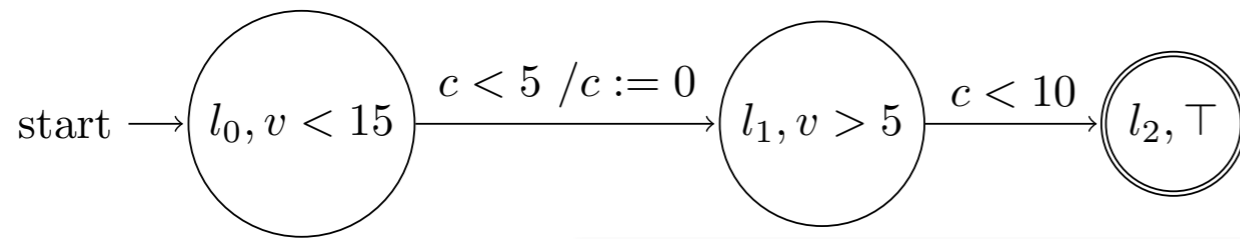


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

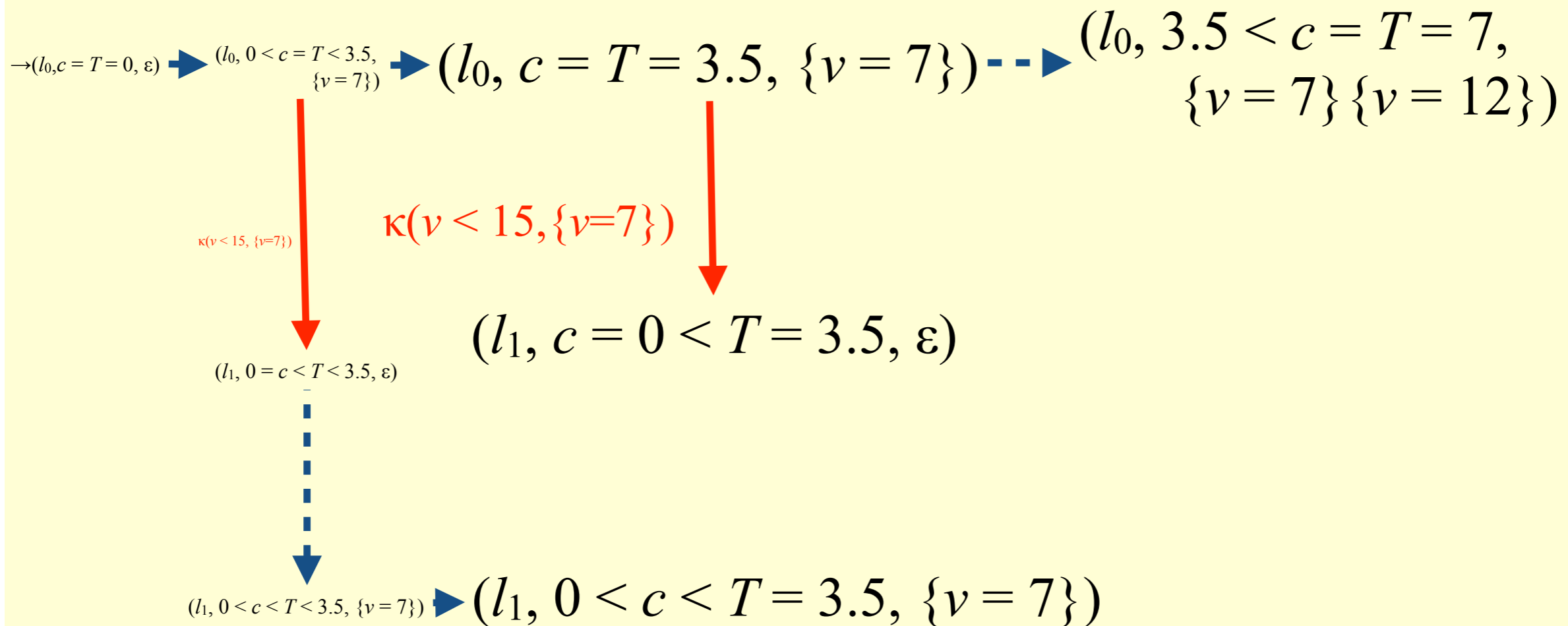
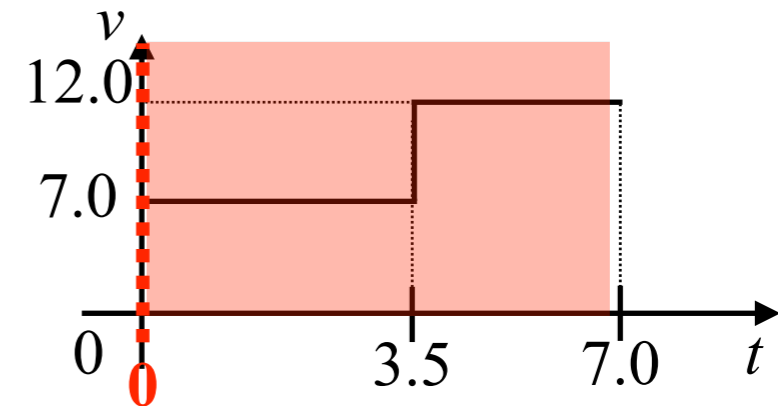


# Zone construction with weight

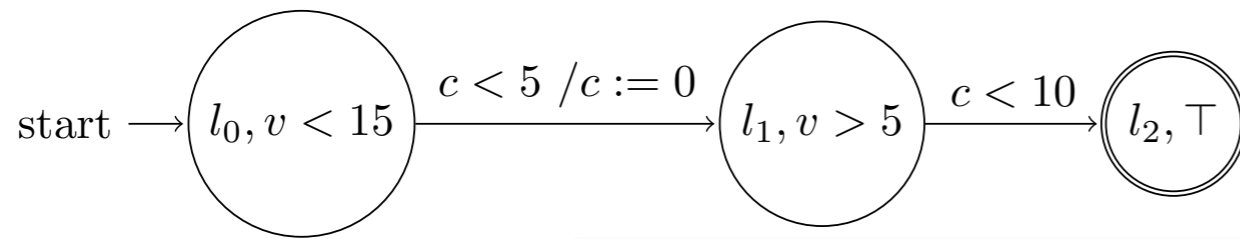


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

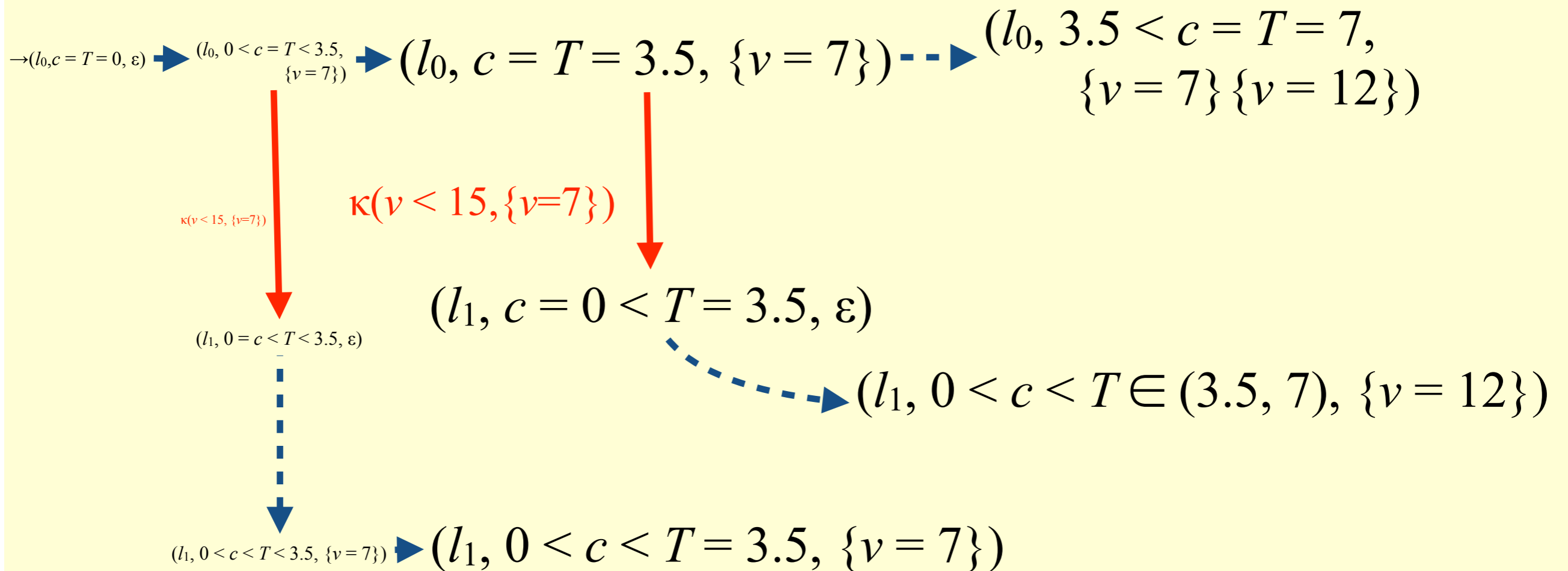
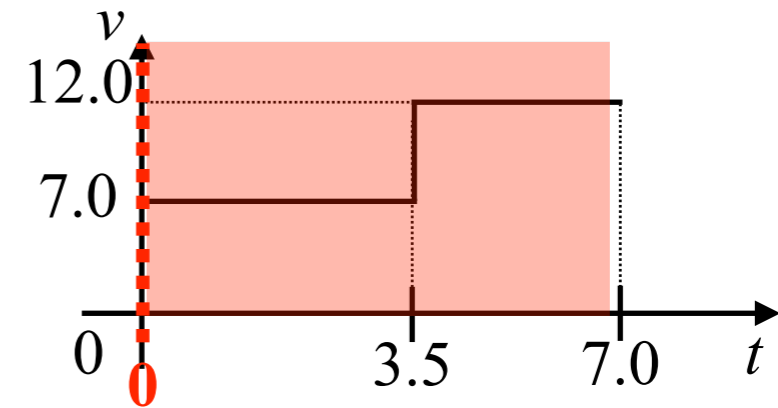


# Zone construction with weight

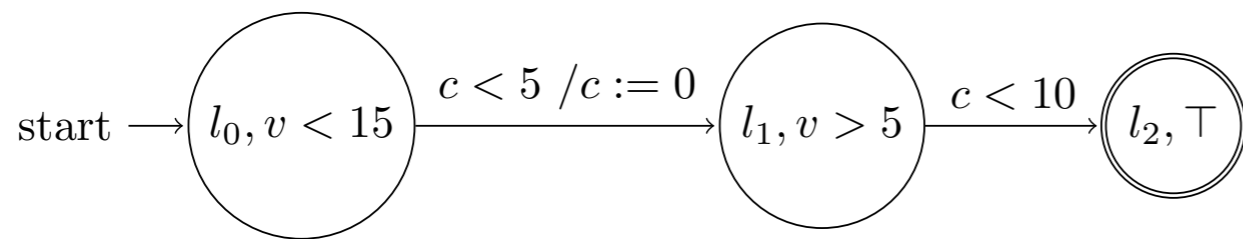


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

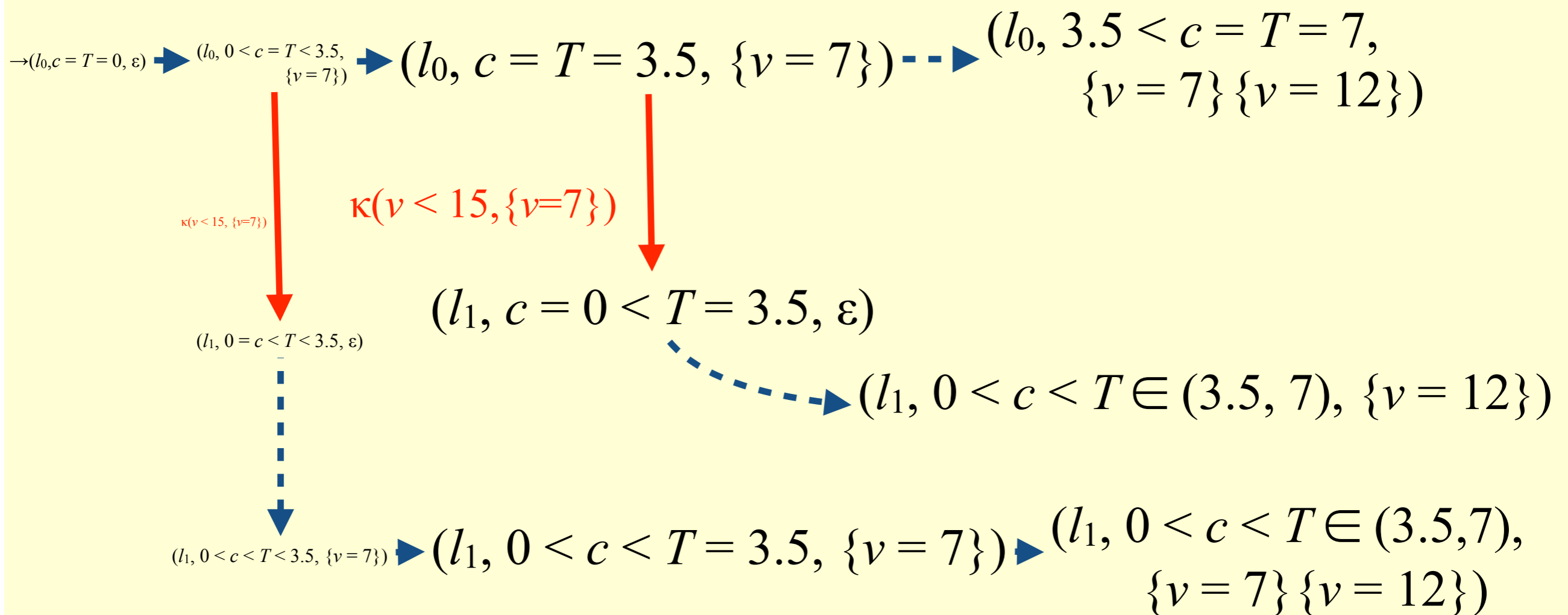
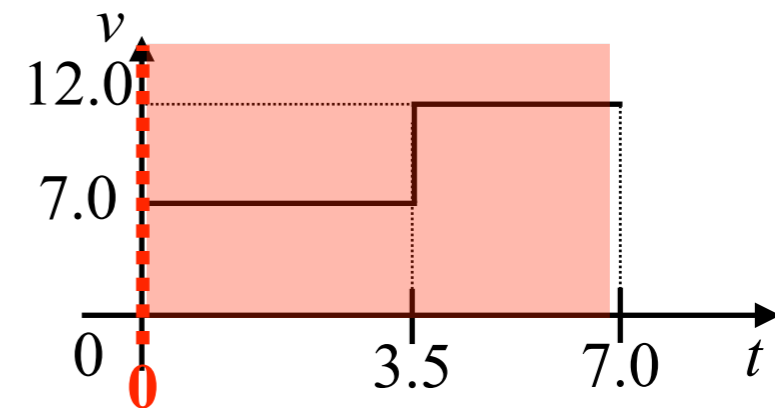


# Zone construction with weight

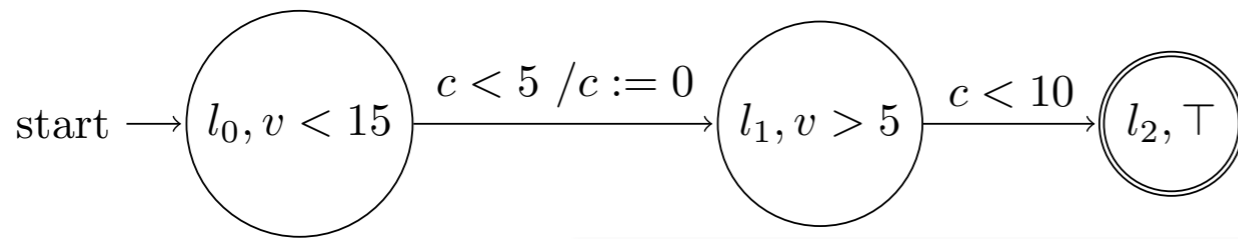


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

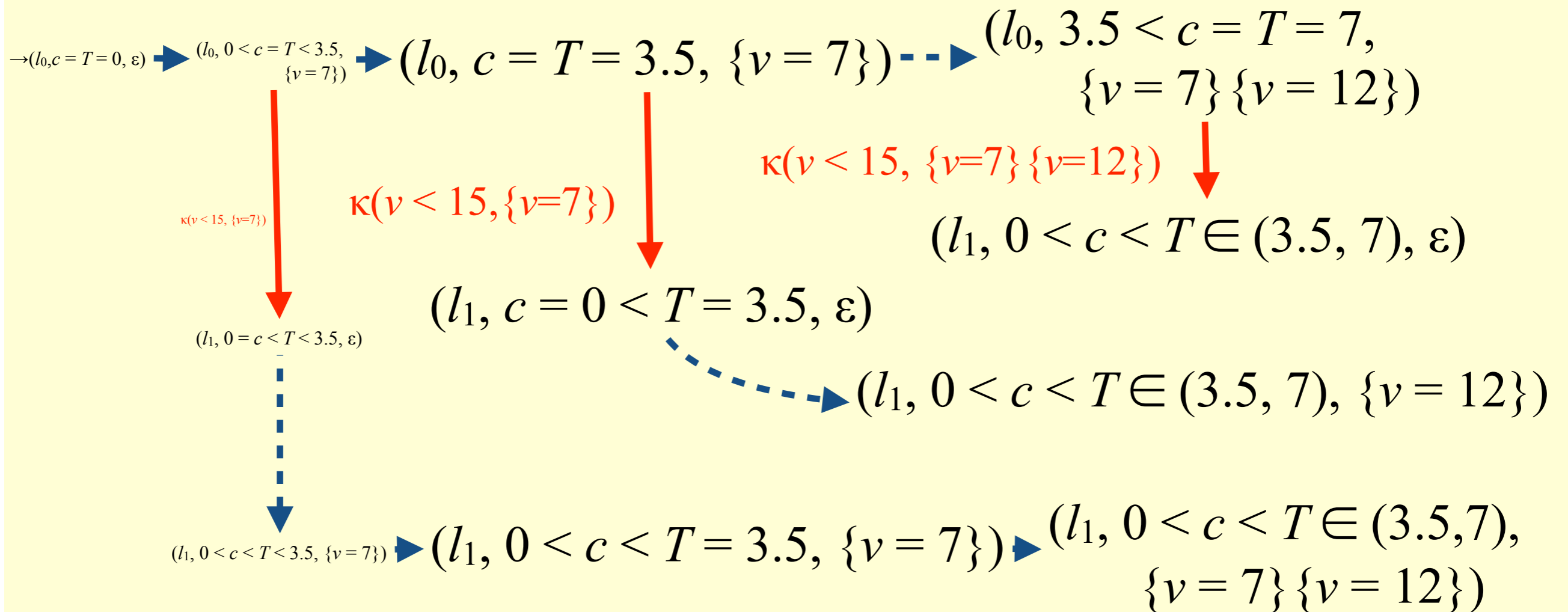
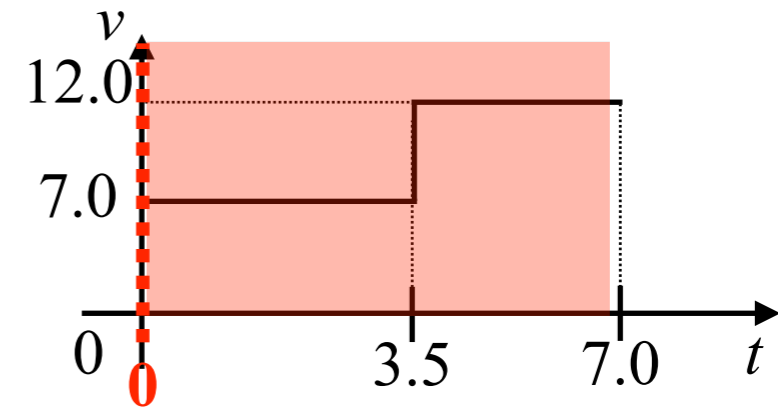


# Zone construction with weight

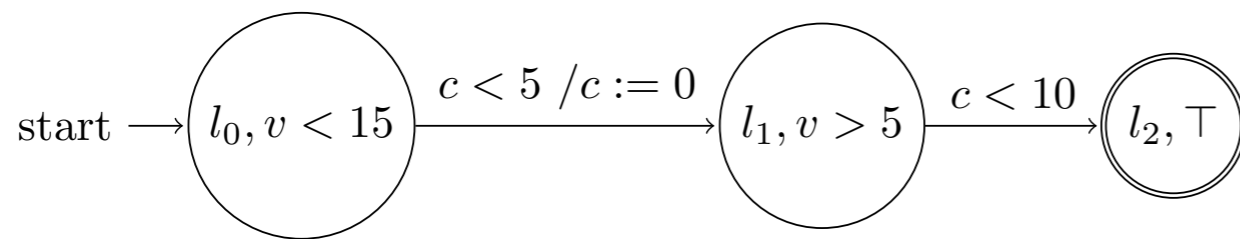


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

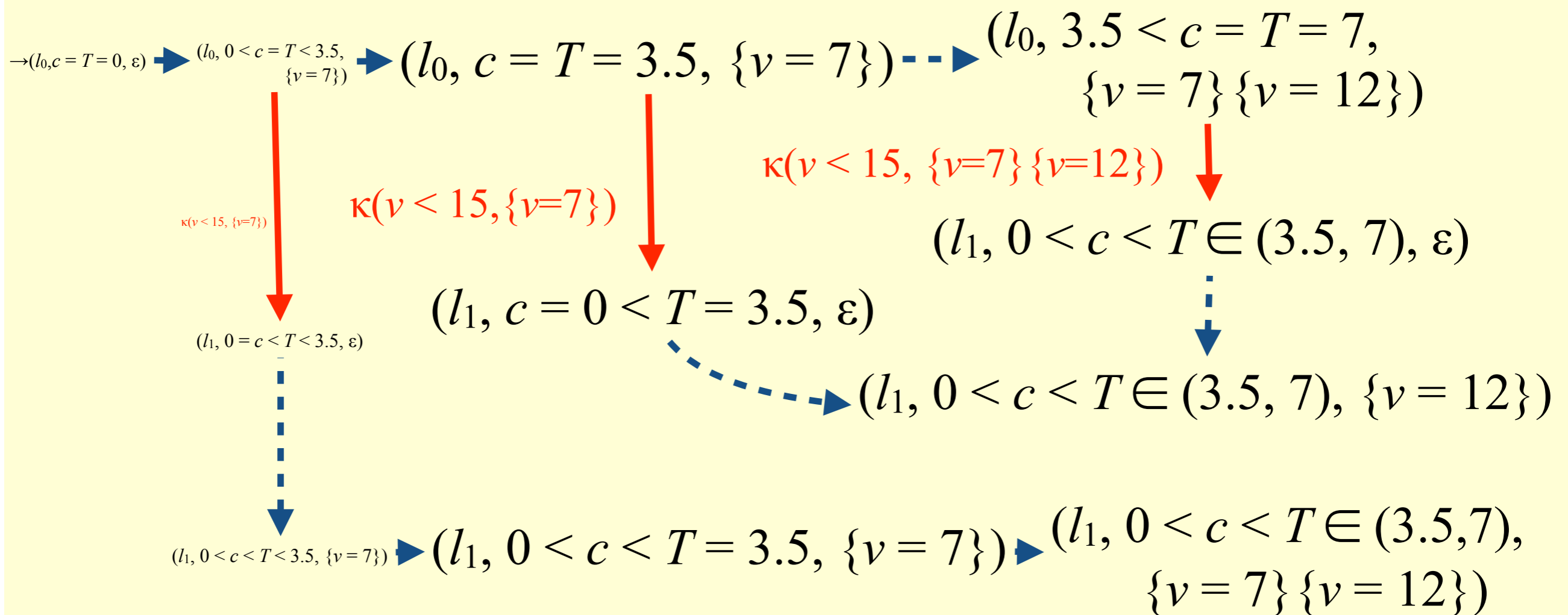
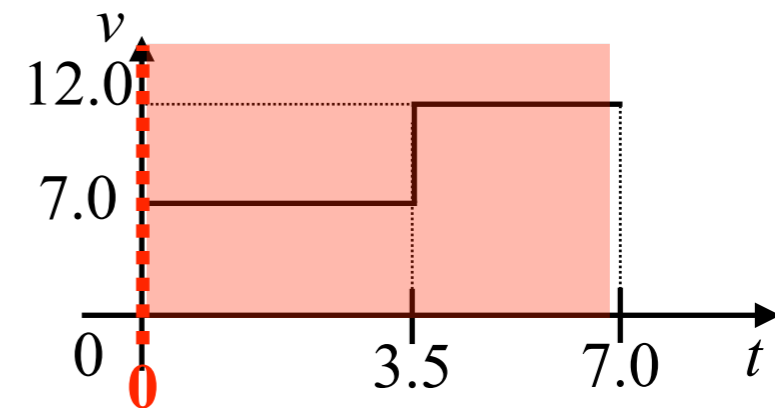


# Zone construction with weight



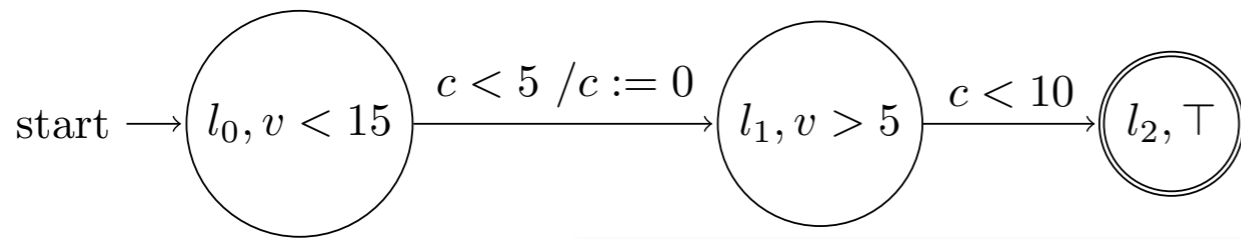
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring



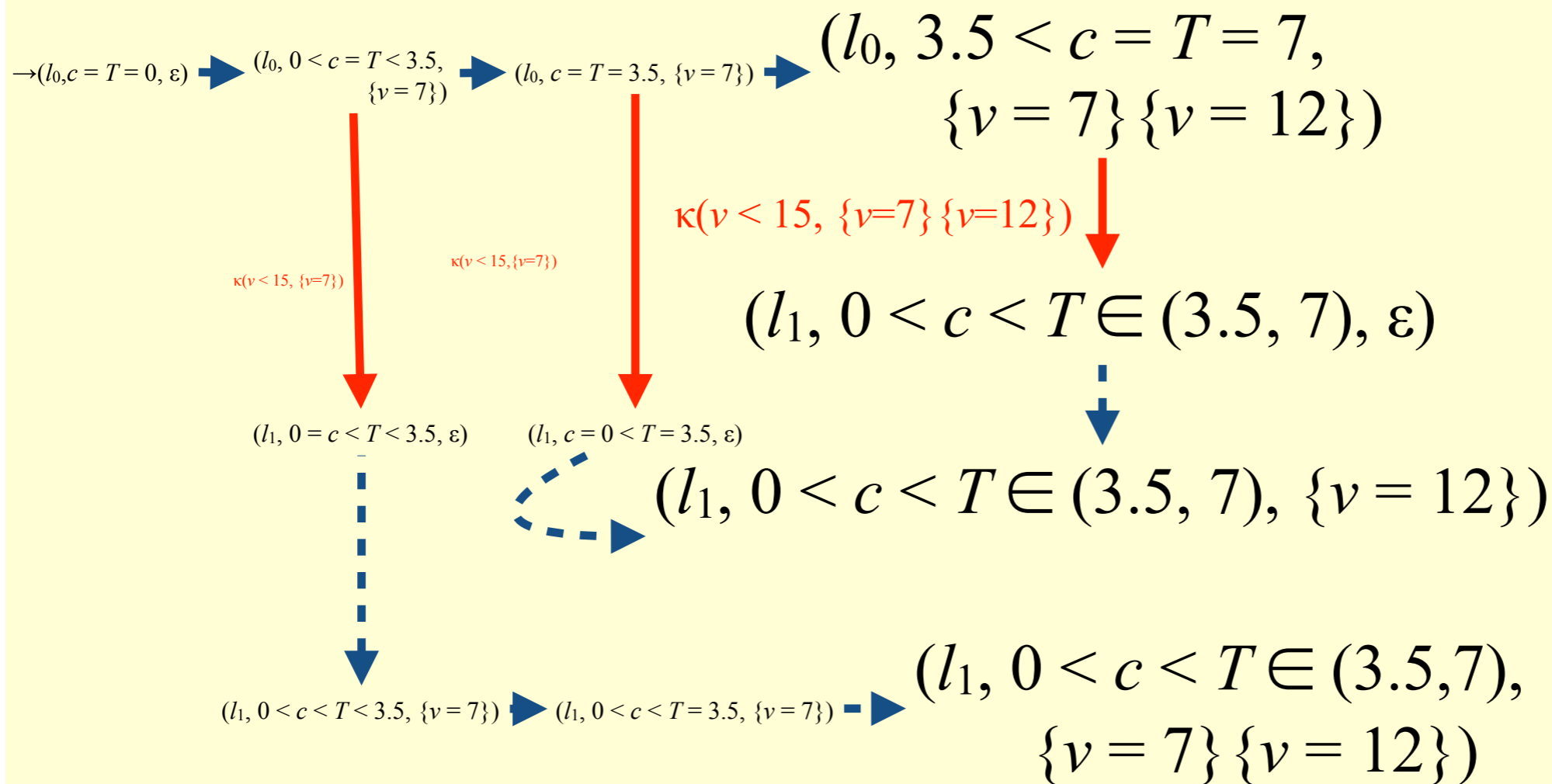
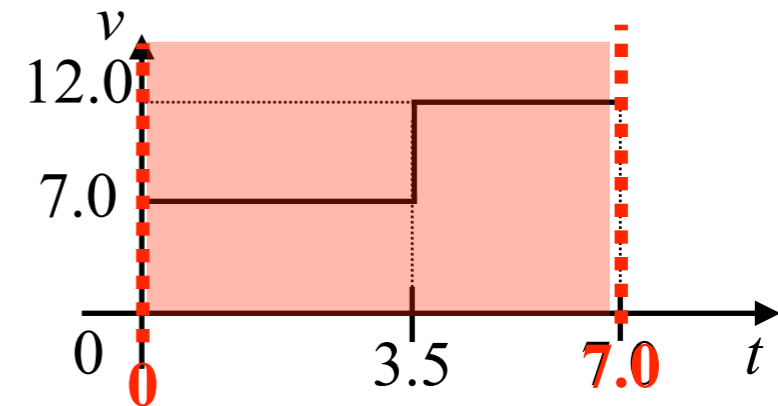


# Zone construction with weight



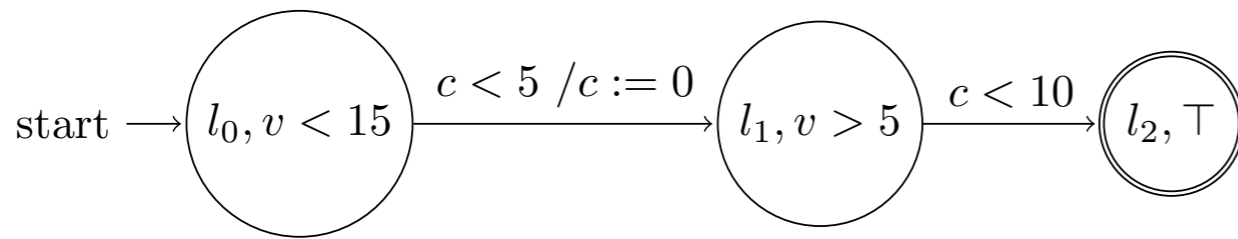
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring



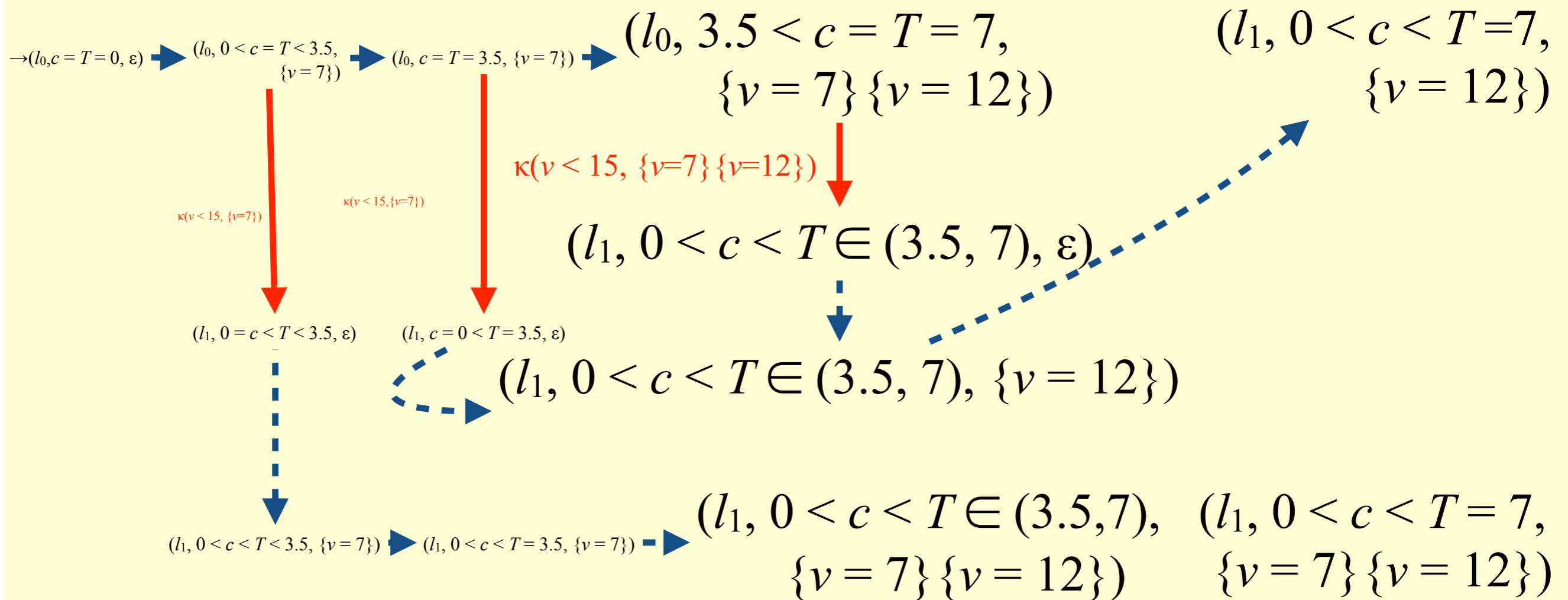
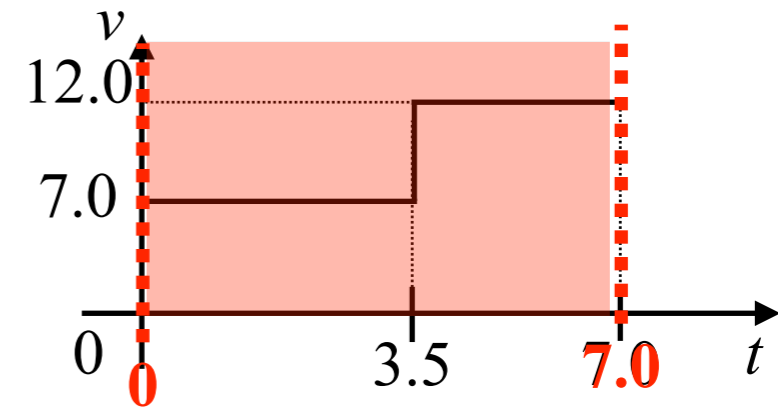


# Zone construction with weight

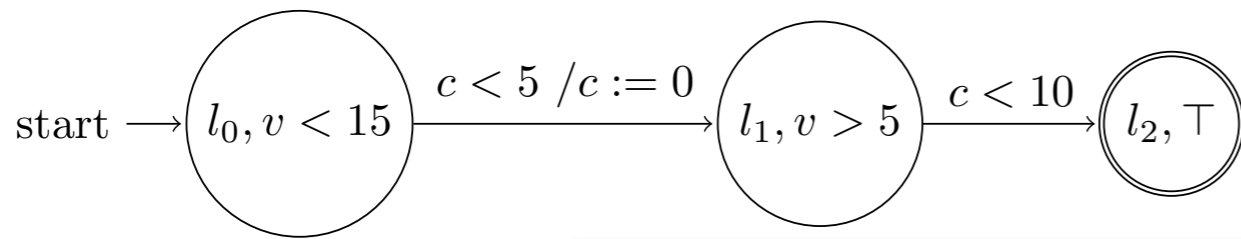


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

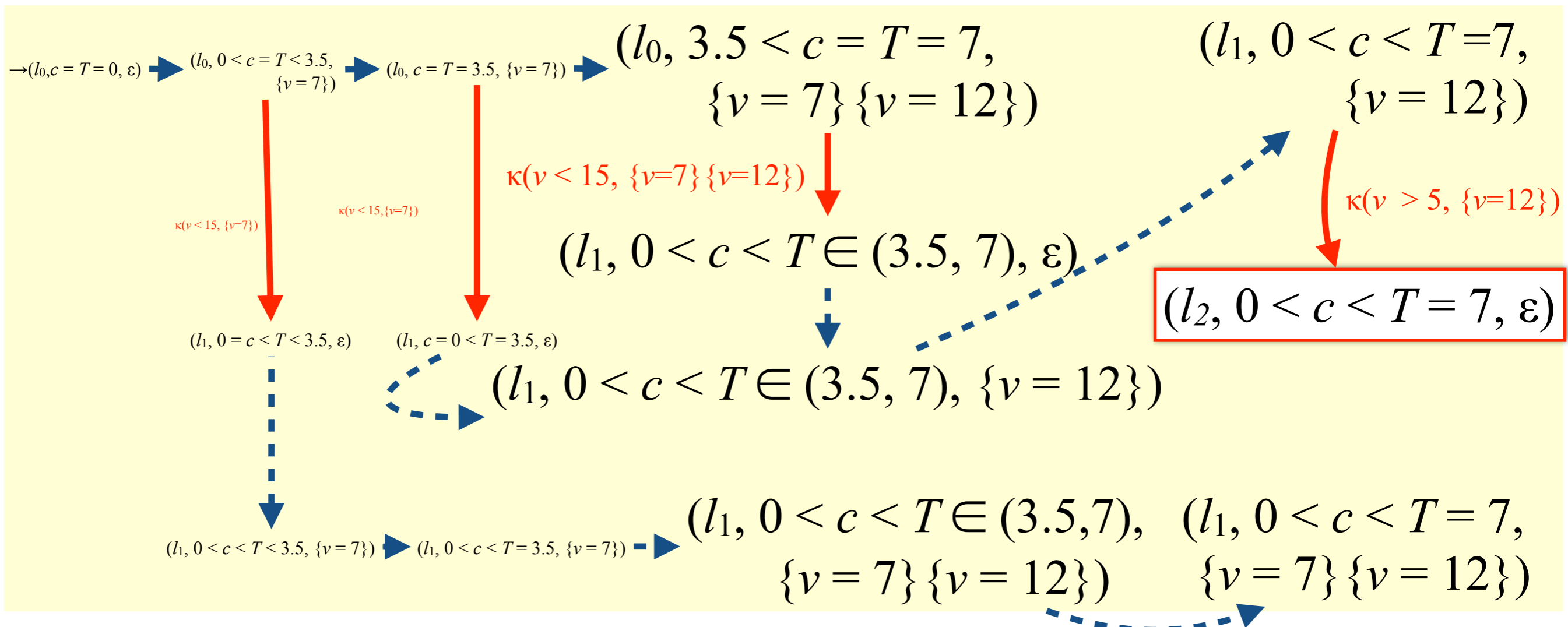
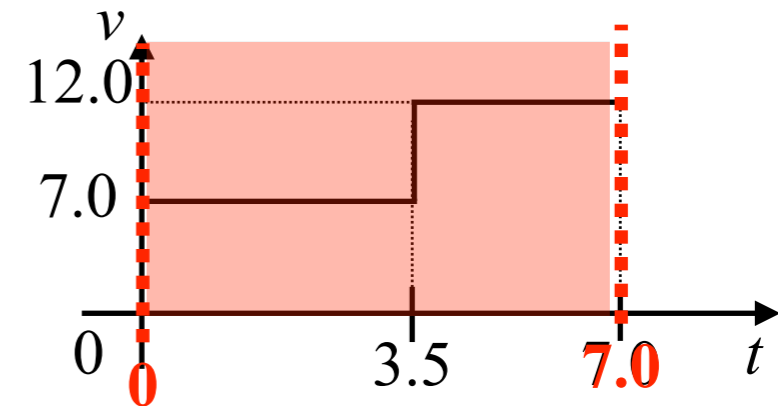


# Zone construction with weight

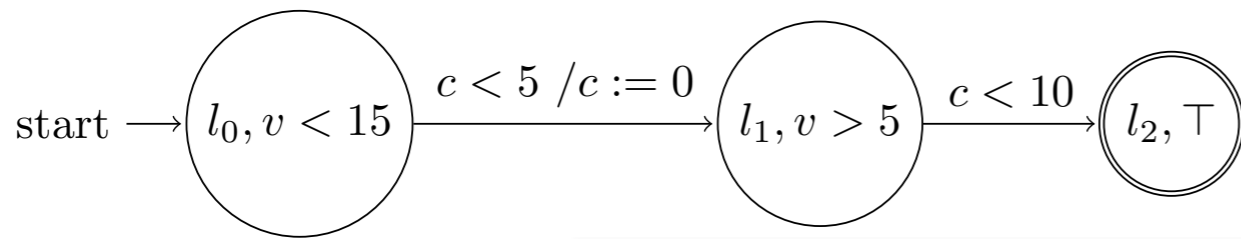


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

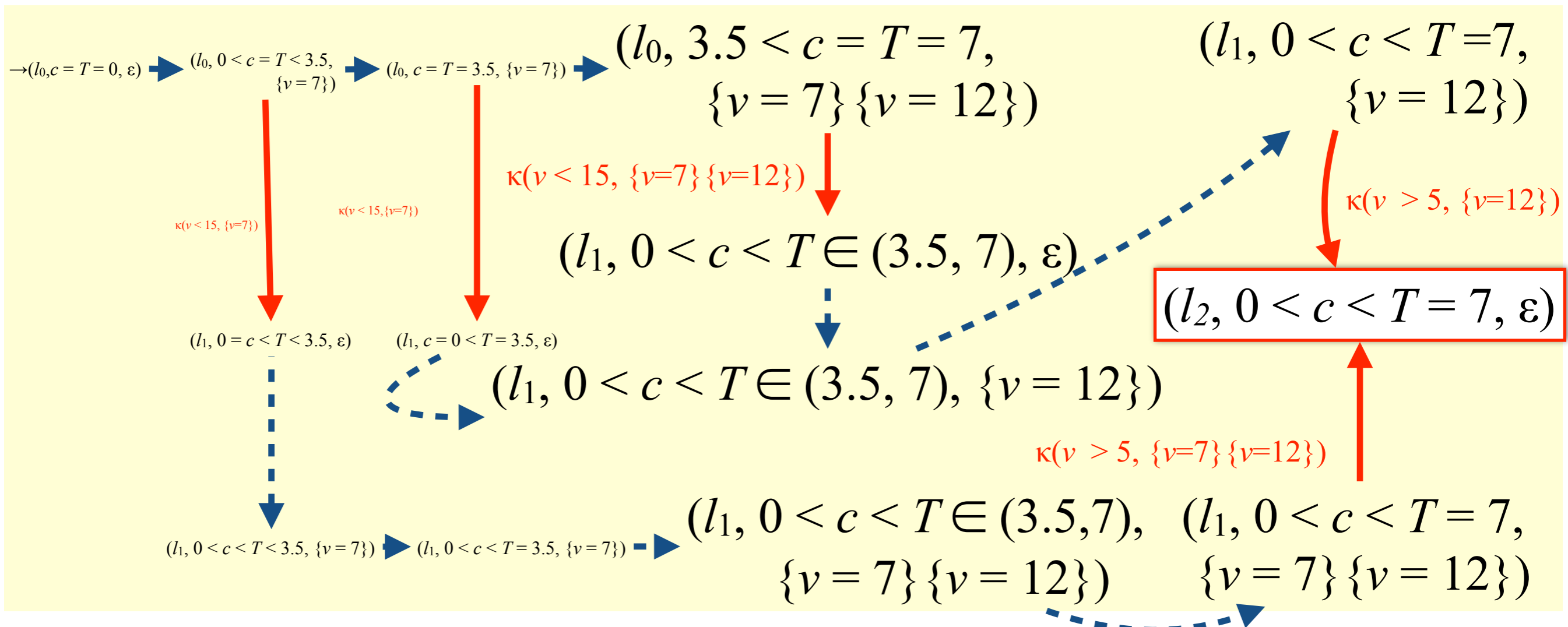
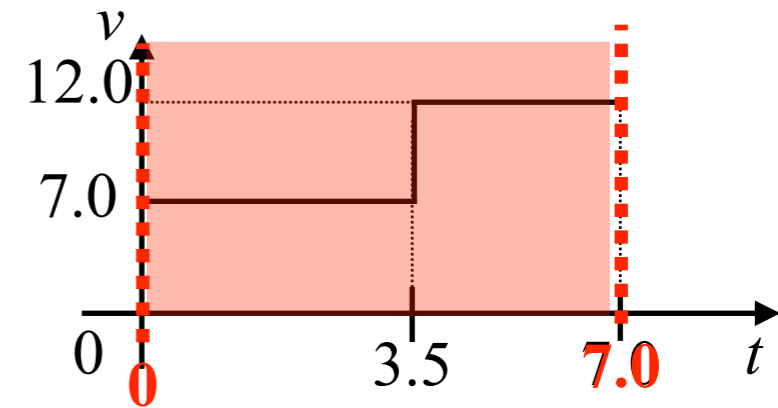


# Zone construction with weight



- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring



# Main Theorem: Correctness

## Thm.

The shortest distance in the zone graph with weight is same as the shortest distance in the weighted TTS for any complete and idempotent semiring.

All of them work!!

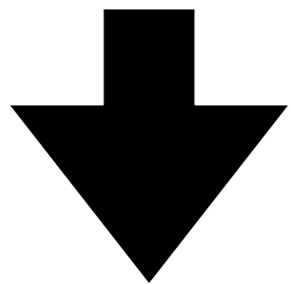
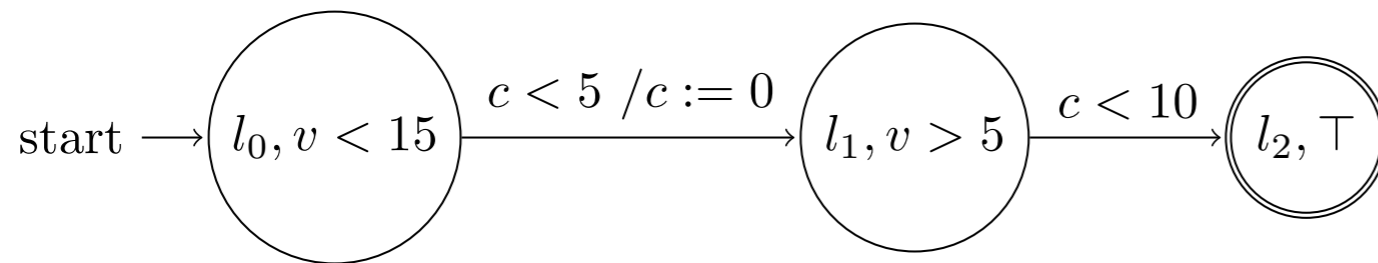
	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	sup	inf
$\otimes$	$\wedge$	inf	+

# Local Conclusion: Zone Construction with Weight

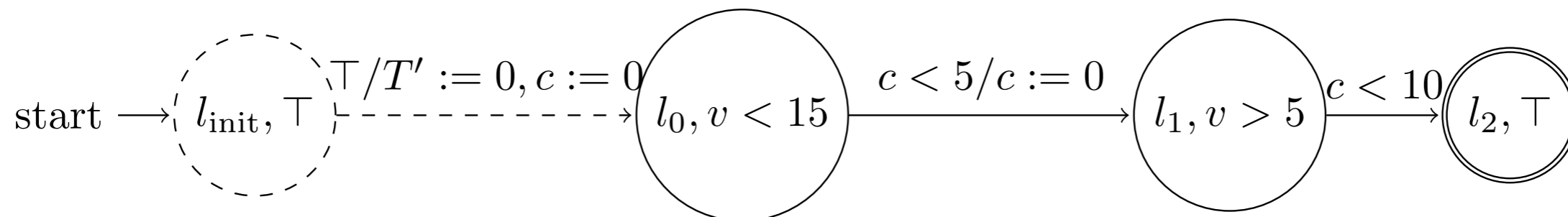
- The construction is basically same as the usual zone construction
- Weights are same as weighted TTS
- The state space is **finite** thanks to **zones** and **finite horizon** of the input signal

# Matching Automata for Pattern Matching

[Bakhirkin+, FORMATS'18]



- Add  $l_{init}$  to wait for the beginning of the matching
- Add clock variable  $T'$  for the beginning of the matching





# Outline

- Motivation + Introduction
- Technical Part
  - Timed symbolic weighted automata (TSWA)
    - TSWA: TA with signal constraints + weight function
  - Quantitative monitoring/timed pattern matching algorithm
    - Idea: zone construction with weight
- Experiments

# Environment of Experiments

- **Semirings:** sup-inf ( $\mathbb{R} \cup \{\pm \infty\}$ , sup, inf) and tropical ( $\mathbb{R} \cup \{+\infty\}$ , inf, +)
- Used 3 original benchmarks (automotive):
  - Inspired by ST-Lib [Kapinski+, SAE Technical Paper'16]

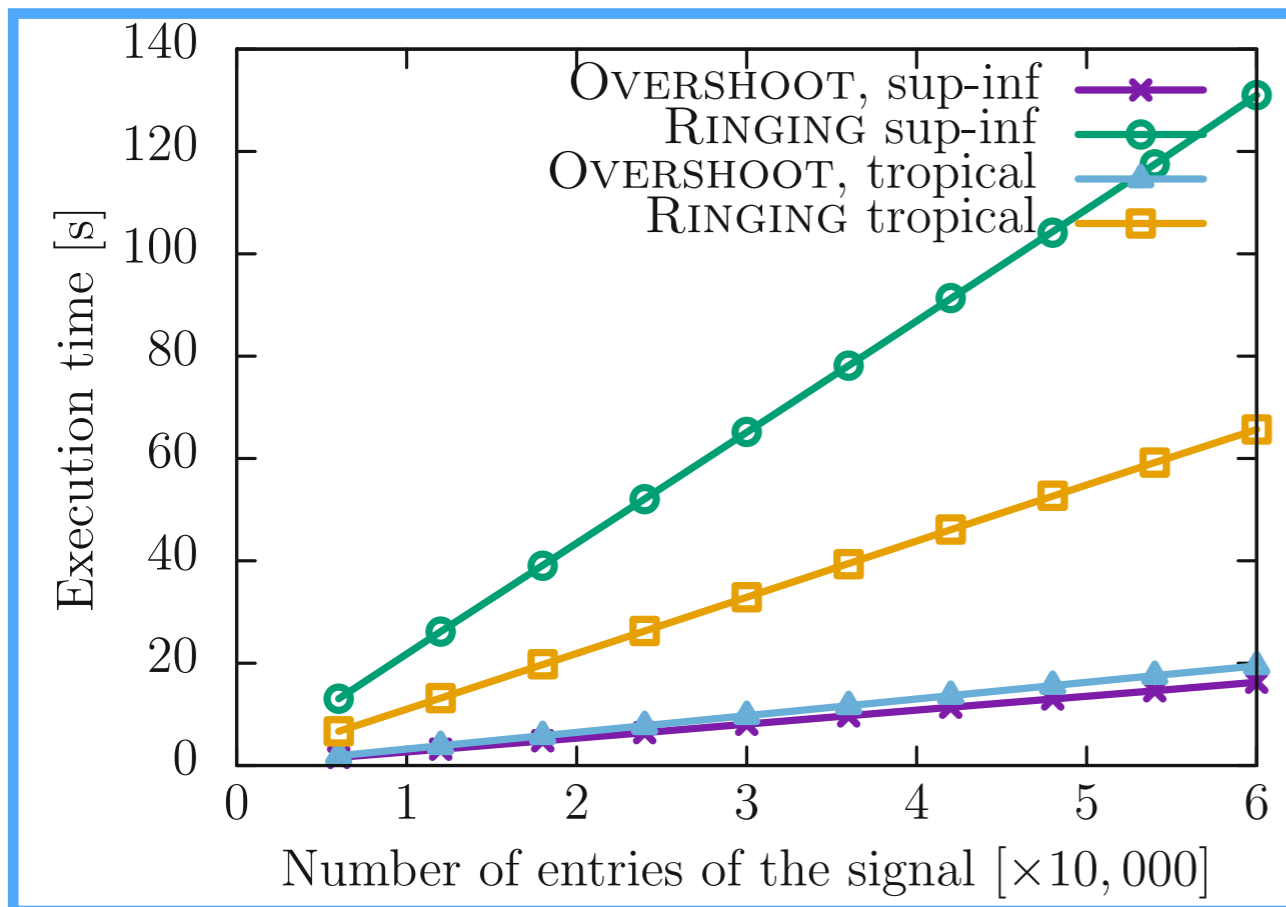
- **Overshoot:**  $|v_{\text{ref}} - v|$  gets large after  $v_{\text{ref}}$  changed
  - Only matches the sub-signals of **length < 150 time units**
- **Ringing:**  $v(t) - v(t-10)$  gets positive and negative repeatedly
  - Only matches the sub-signals of **length < 80 time units**

- **Overshoot (unbounded):**  $|v_{\text{ref}} - v|$  gets large after  $v_{\text{ref}}$  changed
  - No such *bounded*

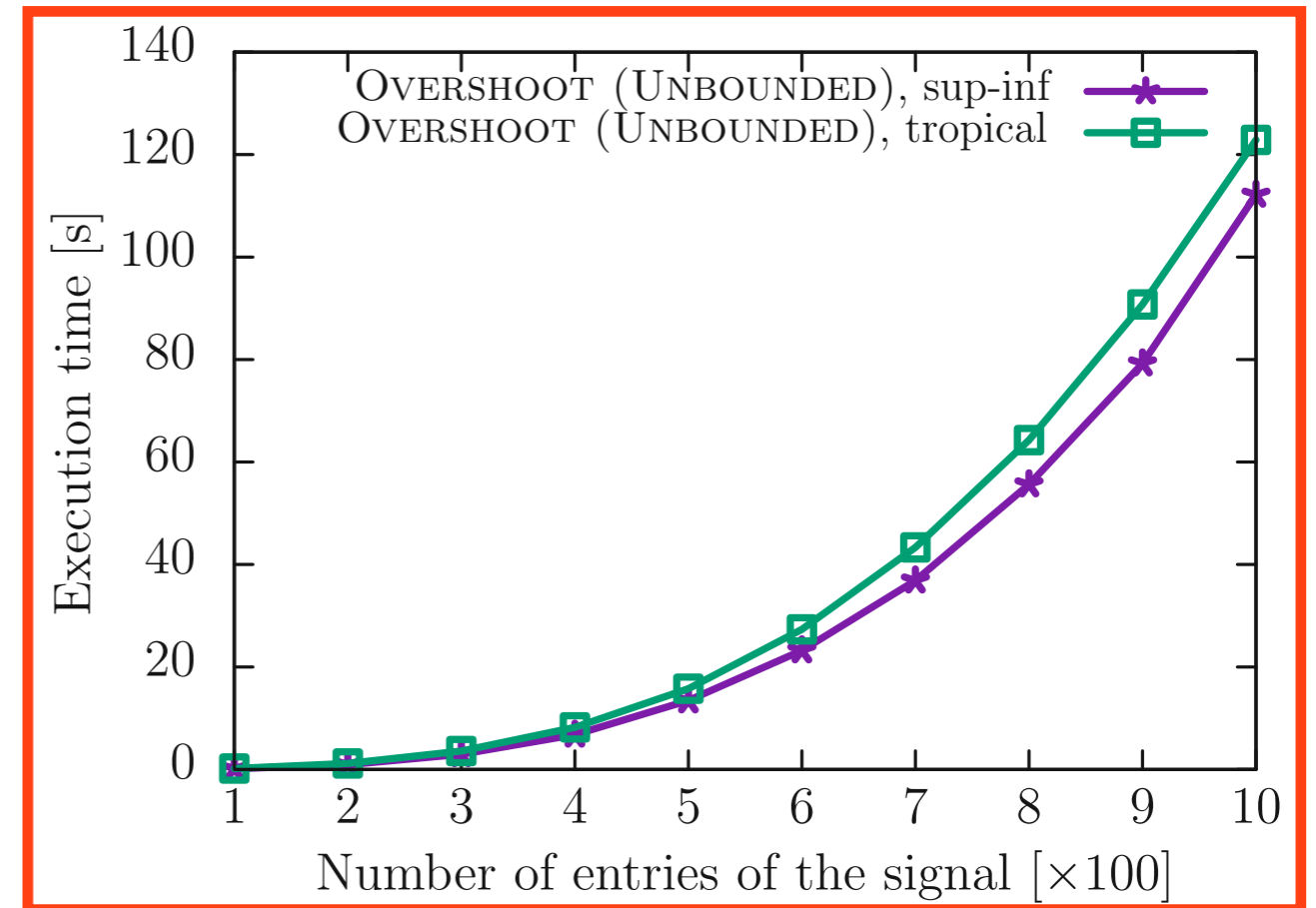
- Amazon EC2 c4.large instance / Ubuntu 18.04 LTS (64 bit)
  - 2.9 GHz Intel Xeon E5-2666 v3, 2 vCPUs, 3.75 GiB RAM

# Execution Time

## Bounded



## Unbounded



- Execution time is **linear** for the bounded spec.
  - 1,000 entries / 1 or 2 sec.
- Execution time explodes for the unbounded spec.

# Conclusion

- Introduced timed symbolic weighted automata (**TSWA**)
  - **TSWA**: TA with signal constraints + weight function
- Gave quantitative monitoring/timed pattern matching algorithm
  - **Idea**: zone construction with weight
- Implementation + experiments
  - **scalable** for bounded specifications

# Appendix

# Example: “Robust” Semantics

**Weight Function:** minimum distance from the threshold

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(u, (a_i))$$

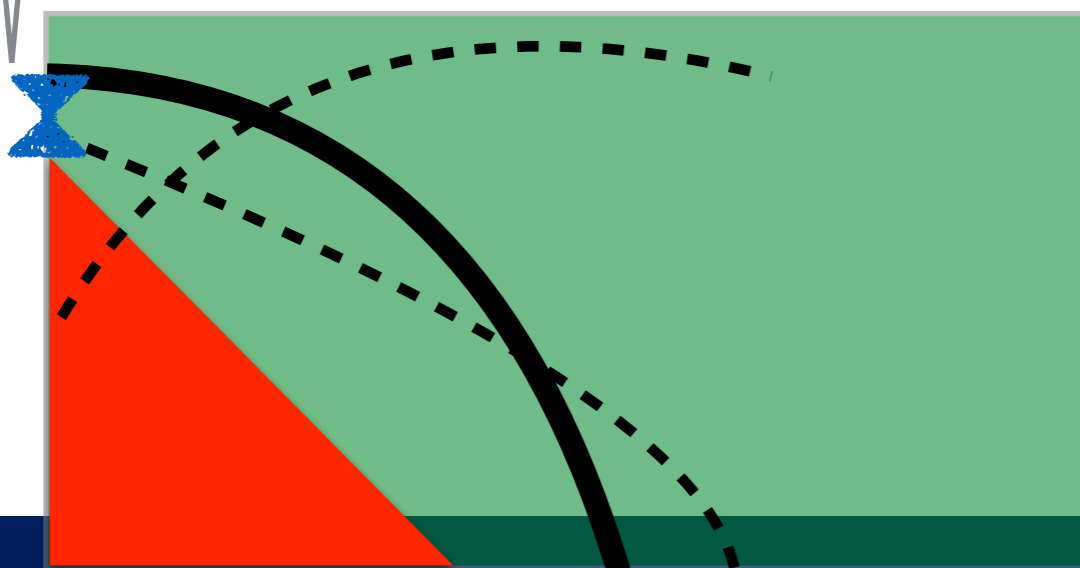
$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \quad \text{where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \quad \text{where } \prec \in \{\leq, <\}$$

Robustness

**Semiring:** sup-inf semiring



# Example: “Robust” Semantics

**Weight Function:** minimum distance from the threshold

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, m\}} \kappa_r(u, (a_i))$$

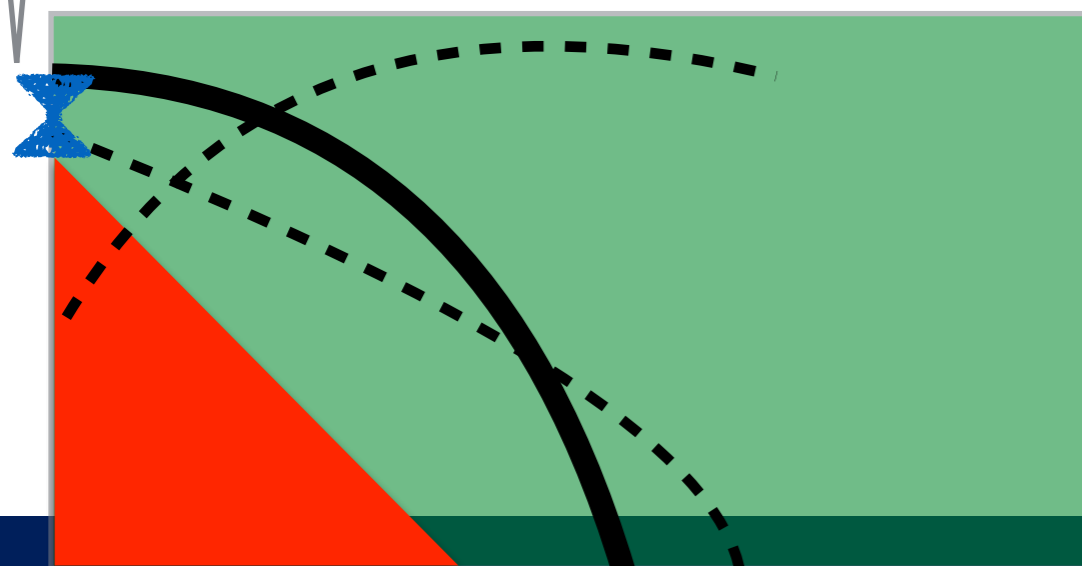
$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \quad \text{where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \quad \text{where } \prec \in \{\leq, <\}$$

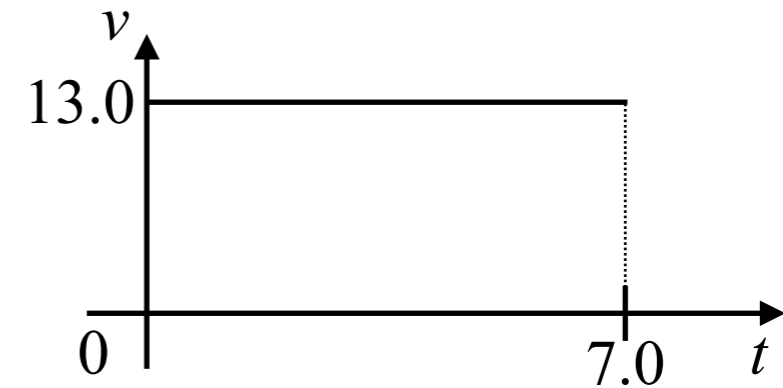
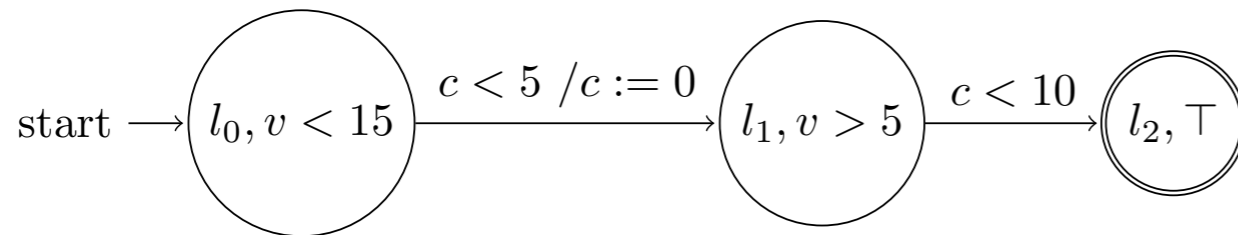
Robustness

**Semiring:** sup-inf semiring



	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	sup	inf
$\otimes$	$\wedge$	inf	+

# Comparison of the semiring



$\rightarrow (l_0, c=0, 0, \varepsilon) \dashrightarrow (l_0, c=2, 2, \{v=7\}) \rightarrow (l_1, c=0, 3, \varepsilon) \dashrightarrow (l_1, c=5, 7, \{v=7\}\{v=12\}) \rightarrow (l_2, c=5, 7, \varepsilon)$

$$\kappa(v < 15, \{v=13\}) = 2 \quad \otimes \quad \kappa(v > 5, \{v=13\}) = 8$$

## Sup-inf semiring

$$2 \otimes 8 = \inf(2, 8) = 2$$

## Tropical semiring

$$2 \otimes 8 = 2 + 8 = 10$$

	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	sup	inf
$\otimes$	$\wedge$	inf	+



# (Qualitative) timed pattern matching

## Input

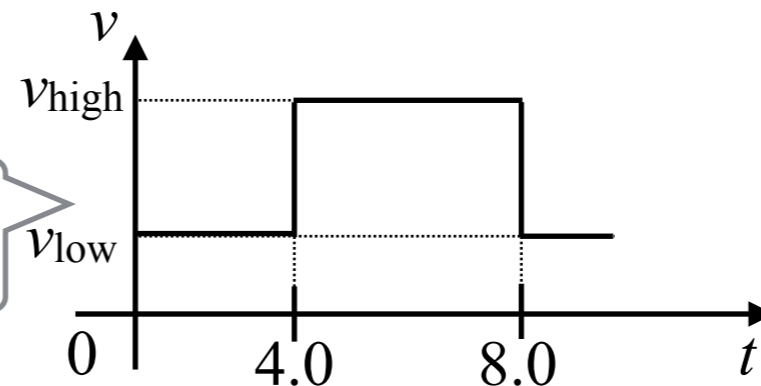
[Ulus+, FORMATS'14]

- **Finite-valued signal**  $\sigma$

- System **log**

discretized!!

- e.g.,



- **Real-time spec.**  $\mathcal{W}$

- **Spec.** to be monitored

- e.g., The velocity should not keep high for  $> 1$  sec.

## Output

- **All** the subsignals  $\sigma([t,t'])$  of the **log** satisfies the **spec.**

- e.g.,  $\sigma([4.0,8.0])$ ,  $\sigma([6.0,8.0])$ ,  $\sigma([6.0,7.5])$ , ...

# (Qualitative) timed pattern matching

## Input

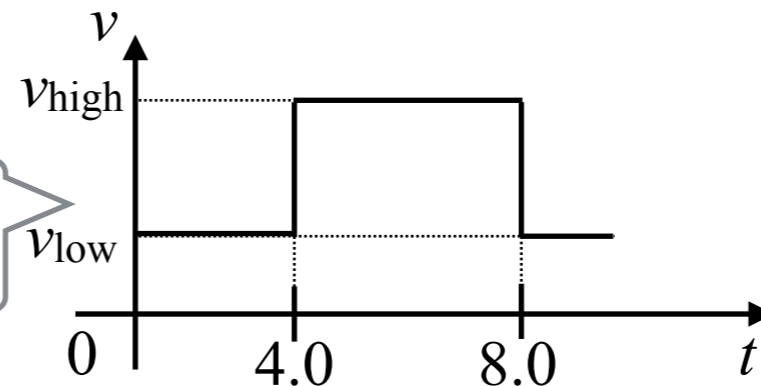
[Ulus+, FORMATS'14]

- **Finite-valued signal**  $\sigma$

- System **log**

discretized!!

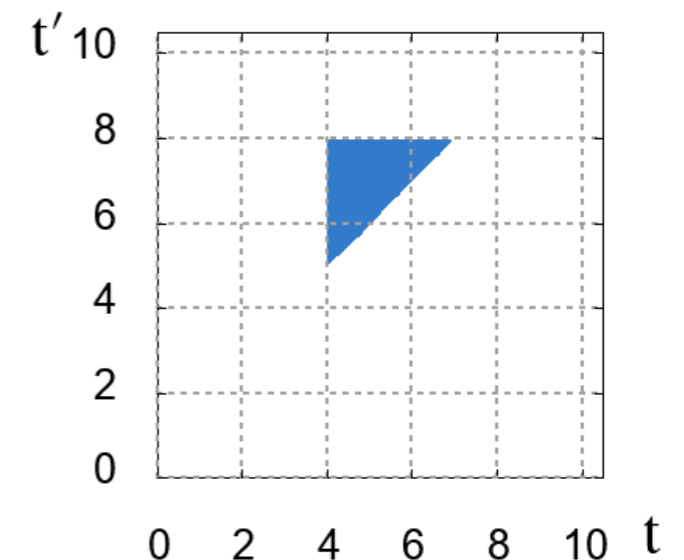
- e.g.,



- **Real-time spec.**  $\mathcal{W}$

- **Spec.** to be monitored

- e.g., The velocity should not keep high f



## Output

- **All** the subsignals  $\sigma([t,t'])$  of the **log** satisfies the **spec.**

- e.g.,  $\sigma([4.0,8.0])$ ,  $\sigma([6.0,8.0])$ ,  $\sigma([6.0,7.5])$ , ...

# (Qualitative) timed pattern matching

## Input

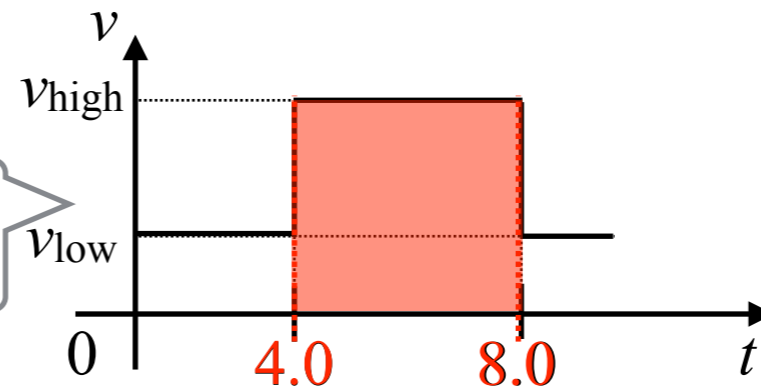
[Ulus+, FORMATS'14]

- **Finite-valued signal  $\sigma$**

- System **log**

discretized!!

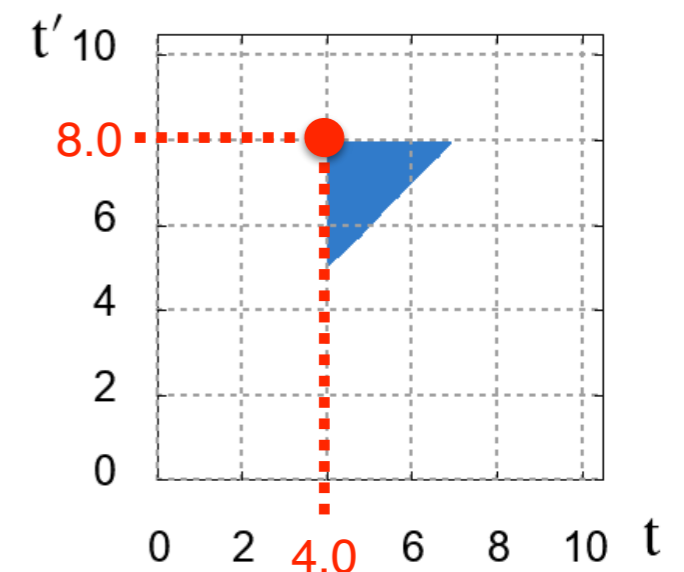
- e.g.,



- **Real-time spec.  $\mathcal{W}$**

- **Spec.** to be monitored

- e.g., The velocity should not keep high f



## Output

- **All** the subsignals  $\sigma([t, t'])$  of the **log** satisfies the **spec.**

- e.g.,  $\sigma([4.0, 8.0])$ ,  $\sigma([6.0, 8.0])$ ,  $\sigma([6.0, 7.5])$ , ...

# (Qualitative) timed pattern matching

## Input

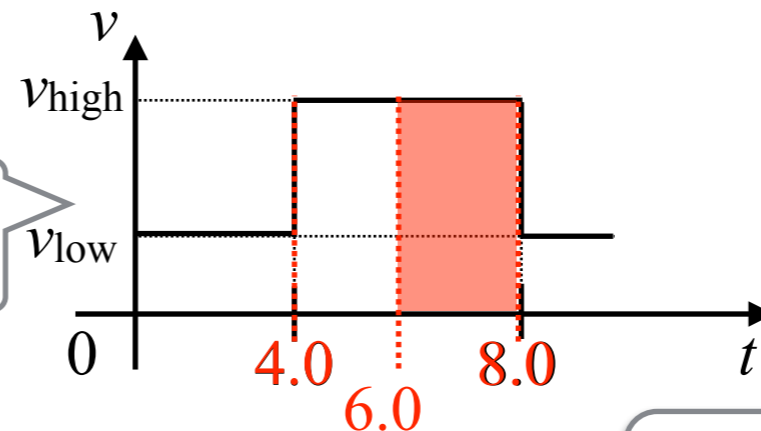
[Ulus+, FORMATS'14]

- **Finite-valued signal**  $\sigma$

- System **log**

discretized!!

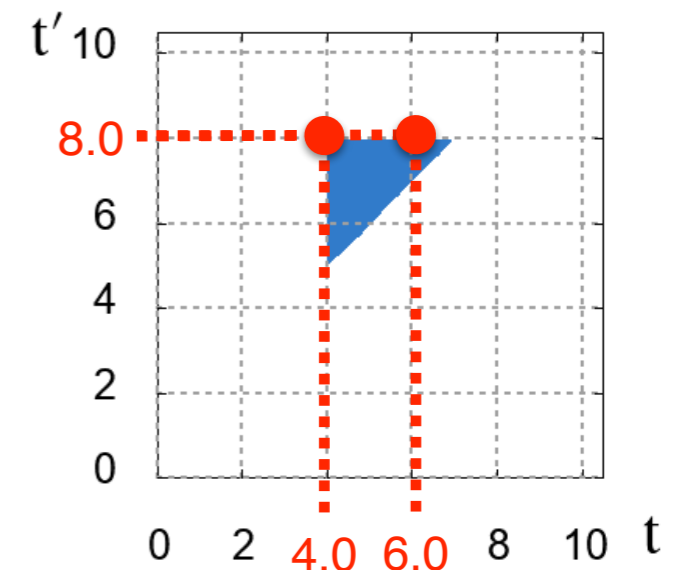
- e.g.,



- **Real-time spec.**  $\mathcal{W}$

- **Spec.** to be monitored

- e.g., The velocity should not keep high f



## Output

- **All** the subsignals  $\sigma([t,t'])$  of the **log** satisfies the **spec.**

- e.g.,  $\sigma([4.0,8.0])$ ,  $\sigma([6.0,8.0])$ ,  $\sigma([6.0,7.5])$ , ...

# (Qualitative) timed pattern matching

## Input

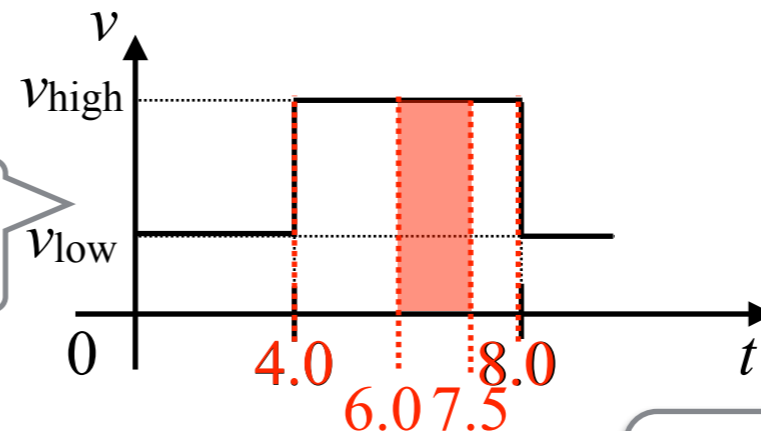
[Ulus+, FORMATS'14]

- **Finite-valued signal**  $\sigma$

- System **log**

discretized!!

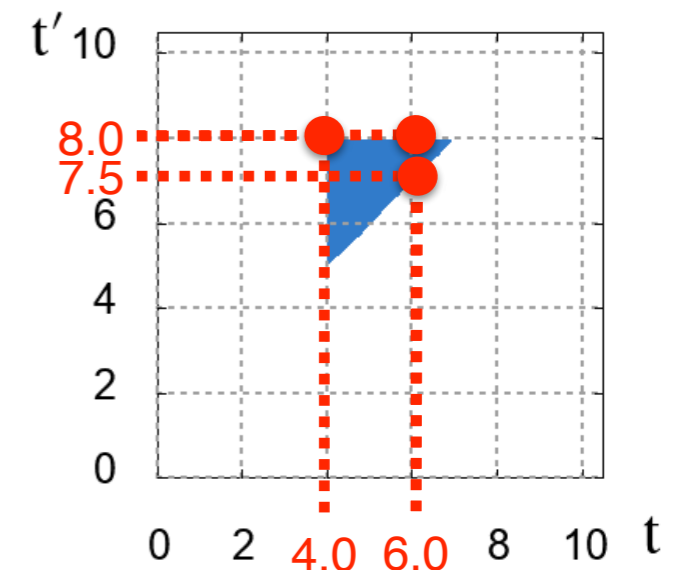
- e.g.,



- **Real-time spec.**  $\mathcal{W}$

- **Spec.** to be monitored

- e.g., The velocity should not keep high f



## Output

- **All** the subsignals  $\sigma([t, t'])$  of the **log** satisfies the **spec.**

- e.g.,  $\sigma([4.0, 8.0])$ ,  $\sigma([6.0, 8.0])$ ,  $\sigma([6.0, 7.5])$ , ...

# (Qualitative) timed pattern matching

## Input

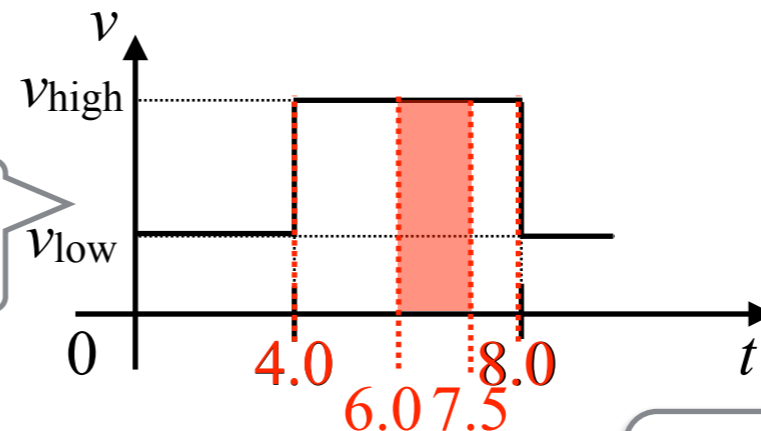
[Ulus+, FORMATS'14]

- **Finite-valued signal**  $\sigma$

- System **log**

discretized!!

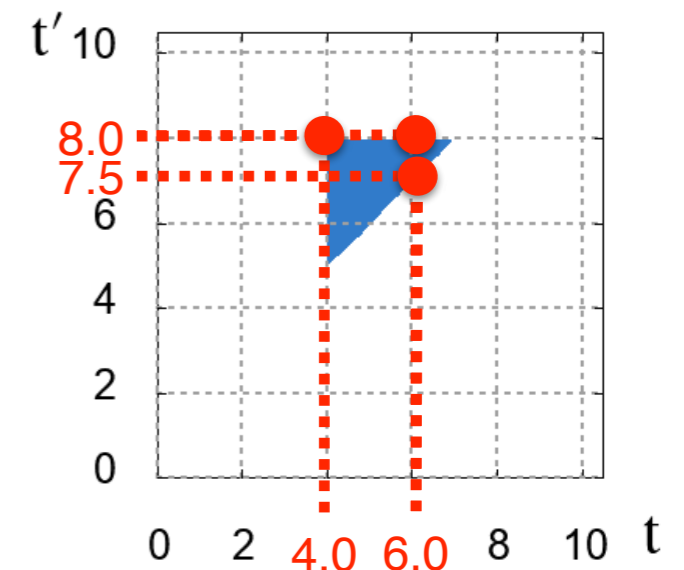
- e.g.,



- **Real-time spec.**  $\mathcal{W}$

- **Spec.** to be monitored

- e.g., The velocity should not keep high f



## Output

- **All** the subsignals  $\sigma([t, t'])$  of the **log** satisfies the **spec.**

- e.g.,  $\sigma([4.0, 8.0])$ ,  $\sigma([6.0, 8.0])$ ,  $\sigma([6.0, 7.5])$ , ...

We want to know **how robustly** the spec. is satisfied!!